U.G. 2nd Semester Examination - 2022

MATHEMATICS

[PROGRAM]

Course Code : BMTMCCRT201

Course Title : Ordinary Differential equations and Linear Algebra

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) Write down the order and degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=1+x$.

b) Eliminate the arbitrary constants A and B from the relation $y=Ae^{x}+Be^{-x}+x^2$.

c) Find an integrating factor of the differential equation $e^{xy}ydx-x\tan^{-1}(xy)dy=0$.

[Turn Over]

d) Find out the orthogonal trajectories of a system of concurrent straight lines $y=mx$ (m is a constant).

e) Form the differential equation whose primitive is $ax+by+c=0$, a, b, c being parameters.

f) For what value of λ, $e^{\lambda x}$ will be trial solution of the equation $\frac{d^2y}{dx^2}+y=0$?

g) Find the value of m which makes the differential equation

$$\left(a^2-mxy-y^2\right)dx-(x+y)^2\,dy=0 \text{ exact.}$$

h) Find the Wronskian of the functions $\sin x,\ \cos x$.

i) Solve: $\left(D^2+4\right)y=0$, $D\equiv\frac{d}{dx}$.

j) What do you mean by the dimension of a vector space?

k) Show that $W=\left\{(x, y, z)\in\mathbb{R}^3 : x+y+z=2\right\}$ is not a vector subspace of $\mathbb{R}^3$.

l) When is the set of vectors $\{\alpha, \beta, \gamma\}$ said to be linearly dependent?

m) Determine the subspace of $\mathbb{R}^3$ spanned by the vectors $\alpha = (1, 2, 3)$, $\beta = (3, 1, 0)$.

n) Give an example of an infinite dimensional vector space.

o) Under which condition the homogeneous system of linear equations $Ax = b$ has a non-trivial solution?

2. Answer any **five** questions: 2×5=10

a) The product of the slope and the ordinate at any point (x, y) of a curve passing through the point (5, 3) is equal to the abscissa at that point. Find the equation of the curve.

b) Find the general solution of the differential equation $y = px + \cos p$; where $p \equiv \frac{d}{dx}$.

c) Find the value of $\frac{1}{D-1}\{xe^x\}$; where $D \equiv \frac{d}{dx}$.

d) Write down the general form of Euler-Cauchy type differential equation.

e) Solve: $1 + y^2 + \left(x - e^{-tan^{-1}y}\right)\frac{dy}{dx} = 0$.

f) Show that the set

$$S = \left\{(x, y, z) \in \mathbb{R}^3 : x + 2y - z = 0,\ 2x - y + 3z = 0\right\}$$

is a subspace of the vector space $\mathbb{R}^3$.

g) In $\mathbb{R}^2$, if $\alpha = (3, 1)$ and $\beta = (2, -1)$, show that $L\{\alpha, \beta\} = \mathbb{R}^2$.

h) Show that the vectors (1, 2, 1), (3, 0, –5) are linearly independent.

3. Answer any **two** questions: 5×2=10

a) Solve: $\frac{adx}{yz(b-c)} = \frac{bdy}{zx(c-a)} = \frac{cdz}{xy(a-b)}$.

b) Show that the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal, where λ is a parameter.

c) If U and W be two subspaces of a vector space V, show that

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W).$$

4. Answer any **one** question: 10×1=10

a) i) Reduce the equation

$$\left(px^2 + y^2\right)(px + y) = (p+1)^2$$

to Clairaut's form by using the substitutions $u = xy$ and $v = x + y$ and hence find its complete primitive.

ii) If a non-null vector space V over a field F is spanned by a linearly dependent set $\{\alpha_1, \alpha_2, ..., \alpha_n\}$, then prove that V can also be spanned by a suitable proper subset of S.

iii) Find out the solution of the system of homogeneous equation $AX = O$, where

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \end{pmatrix} \text{ and } X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

5+3+2

b) i) Solve

$$z^2 dx + \left(z^2 - 2yz\right) dy + \left(2y^2 - yz - zx\right) dz = 0.$$

ii) Solve by the method of variation of parameters: $\dfrac{d^2 y}{dx^2} + y = \cos x$. 5+5

c) i) Solve the simultaneous linear equations

$$\frac{dx}{dt} = 5x + 4y, \quad \frac{dy}{dt} = -x + y.$$

ii) Find a basis for $\mathbb{R}^3$ that contains the vector (1, 2, 0) and (1, 3, 1). 5+5

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