U.G. 4th Semester Examination - 2022

MATHEMATICS

[PROGRAM]

Course Code : BMTMCCRT401

Course Title : Partial Differential Equation, Laplace Transform & Tensor Analysis

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Find the order of the partial differential equation $x^2\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=(x-y)^2$.

b) Find the partial differential equation by elimination of 'a', 'b' from $z=(x+a)(y+b)$.

c) State quotient law for tensor of type (1, 0).

d) Define singular integral of a partial differential equation.

e) Prove that $\delta_i^i = n$.

f) What is the complete integral of $p^2+q^2=n^2$?

g) What is the value of $L\{\cos at\}$?

h) Define curl of a vector.

i) Write down the general form of quasi-linear partial differential equation (PDE).

j) Give an example of semi-linear PDE.

k) State sufficient condition for existence of Laplace transform.

l) What will be the value of $L^{-1}\left\{\frac{s}{s^2+\lambda^2}\right\}$?

m) Define Covariant Tensors of order two.

n) State convolution theorem.

o) Show that, Christoffel symbol of second kind is symmetric in second two indices.

2. Answer any **five** questions: 2×5=10

a) Show that, $z=ae^{bx}\sin by$ is a solution of $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$, a, b being arbitrary constants.

b) Form the partial differential equation by eliminating the functions 'f' and 'F' from

$$y=f(x-at)+F(at+x)$$

c) Find $L^{-1}\left\{\frac{2s-5}{s^2-9}\right\}$.

d) Show that $L\left\{e^{at}\right\}=\frac{1}{p-a}$.

e) Prove that $\begin{Bmatrix} p \\ i\ j \end{Bmatrix}=\begin{Bmatrix} p \\ j\ i \end{Bmatrix}$.

f) Prove that $\delta^i_j A^j = A^i$.

g) Write down the charpits auxilary equation for the PDE $yzp^2-q=0$.

h) If $g_{ij}=0$ for $i\neq j$, prove that $g^{ii}=\frac{1}{g_{ii}}$ (no summation over 'i').

3. Answer any **two** questions: 5×2=10

a) Solve: $z^2-pz+qz+(x+y)^2=0$.

b) Calculate the Christoffel's symbols [12, 2] and $\begin{Bmatrix} 2 \\ 12 \end{Bmatrix}$ in a 3-dimensional Riemann space in which the line element ds is given by

$$ds^2=\left(dx^1\right)^2+\left(x^1\right)^2\left(dx^2\right)^2+\left(dx^3\right)^2.$$

c) Apply Convolution theorem to evaluate

$$L^{-1}\left\{\frac{s}{\left(s^2+2s\right)^2}\right\}.$$

4. Answer any **one** question: 10×1=10

a) i) Using Laplace transform solve $y''+2y'+y=3te^{-t}$, given that $y(0)=4$, $y'(0)=2$.

ii) The components of contravariant vector in x-coordinate system are 6 and 3. Obtain its component in $\overline{x}$-coordinate system if $\left(\overline{x}^1\right)=\left(7x^1\right)^2$, $\overline{x}^2=6\left(x^1\right)^2+2\left(x^2\right)^2$.

5+5=10

b) i) Solve using Laplace transform:

$$\left(D^2+m^2\right)y=a\cos nt, \text{ where } D\equiv\frac{d}{dt},$$

and given y=Dy=0, when t=0.

ii) Prove that if A^{ij} is a symmetric tensor then $A^j_{i,j}=\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^j}\left(A^j_i\sqrt{g}\right)-\frac{1}{2}A^{jk}\frac{\partial g_{jk}}{\partial x^i}$.

5+5=10

c) i) Show that, $\frac{\partial g_{ij}}{\partial x^k} - \frac{\partial g_{ik}}{\partial x^j} = [j\ k,\ i] - [i\ j,\ k]$.

ii) If $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}} e^{-\frac{1}{4s}}$, show that

$L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\left(\frac{\pi}{s}\right)}.e^{-\frac{1}{4s}}$. 5+5=10

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