U.G. 4th Semester Examination - 2022

PHYSICS

[HONOURS]

Course Code : BPHSCCHC 401

Course Title : Mathematical Physics III

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

a) Write down CR- equations in polar form.

b) Write the complex number $z=-1-i\sqrt{3}$ in polar form.

c) Check whether the following functions is analytic or not : $e^{x}\left(\cos x+i\sin x\right)$.

d) Find the value of i^{i}.

e) What do you mean by a branch point?

f) Evaluate $\int_{c}(z-a)^{n}dz$ where c is the circle $|z-a|=r$ and n is any integer $\neq -1$.

g) Find the singularities of the function, $f(z)=\frac{3z-2}{(z-1)^{2}(z+1)(z-4)}$.

h) Find the Residue of $\frac{1}{(z-1)^{2}}$ at its singularity.

i) What is nature of a Gaussian function after Fourier transform?

j) What is the output of the integral $\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-ik(x-y)}dk$.

k) Find the Fourier transform of

$$f(x)=1 \text{ for } |x|<1$$
$$=0 \text{ for } |x|>1.$$

l) Show that all the diagonal elements of a skew-Hermitian matrix are either zero or pure imaginary.

m) Show that the inverse of a matrix is unique.

n) What is the value of the determinant of an orthogonal matrix?

o) Give an example of a symmetric matrix.

2. Answer any **five** questions: 2×5=10

a) Solve $x^5 = 1$.

b) Find the Taylor series expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point z = 1.

c) Evaluate $\int_{1-i}^{2+i} (2x+iy+1)\,dz$ along the straight line joining the points (1-i) and (2+i).

d) Evaluate: $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where the contour C is the circle within |z|=3.

e) Show that the matrix $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is Hermitian.

f) If $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ find the value of A^8.

g) Show that the trace of the product of two matrices A and B is independent of the order of the multiplication.

h) If λ is an eigenvalue of an orthogonal matrix, then prove that $\frac{1}{\lambda}$ is also its eigenvalue.

3. Answer any **two** questions: 5×2=10

a) Find the eigenvalues and normalized eigen vector of the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Also check whether A is Hermitian Or not. 4+1

b) Applying the calculus of residue, prove that

$$\int_0^{2\pi} \frac{d\theta}{1-2p\sin\theta+p^2} = \frac{2\pi}{1-p^2}(0<p<1)$$

c) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2, |x| \le 1 \\ 0, \quad |x| > 1 \end{cases}.$$

Hence evaluate $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3}\cos\frac{x}{2}dx$.

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