U.G. 1st Semester Examination - 2021

MATHEMATICS

Course Code : BMTMCCRT101

Course Title : Calculus & Analytical Geometry (2D)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions from the following: $1\times10=10$

a) If $y = e^{\sin x}$, then find $\frac{d^2y}{dx^2}$ at $x=\frac{\pi}{2}$.

b) Show that the function $y = A\cos kx + B\sin kx$, satisfies the equation $\frac{dy^2}{dx^2}+k^2y=0$, where A, B, k are constants.

c) If $z=\log\left(x^2+y^2-1\right)$, then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

d) If $f(x,y)=\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then show that f(x, y) is a homogeneous function.

e) Find the centre and radius of the circle $2x^2+2y^2-4x-2y-5=0$.

f) Transform the equation $\theta=\frac{\pi}{4}$ to Cartesian equation.

g) Give the names of a non-central conic and a central conic.

h) Evaluate: $\int_0^{\frac{\pi}{2}}\cos^3 x\,dx$.

i) Show that the area between the curves $y=x^2$ and $x=y^2$ is $\frac{1}{3}$ sq. units.

j) State Leibnitz's theorem for n-th derivative of the product of two functions.

k) Find the equation of tangent to the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ at (1, 2).

l) Find the point on the conic $\frac{5}{r}=1+3\cos\theta$ whose vectorial angle is $\frac{\pi}{3}$.

m) Evaluate $\lim_{x\to 0}\left\{\cot x.\log\left(\frac{1+x}{1-x}\right)\right\}$.

n) Name the conic $3x^2+2xy+3y^2-16x+20=0$ by finding Δ and D.

o) Find the length of the subtangent of the curve $\log y = x\log a$.

2. Answer any **five** questions: 2×5=10

a) If $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta+\theta\cos\theta)$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$.

b) Find the angle between the curves $y-x^2$ and $y=2-x^2$ at the point of intersection.

c) Find the radius of curvature to the curve $y=e^x$ at the point where it crosses the y-axis.

d) The eccentricities of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$ are e and e' respectively, then show that $\frac{1}{e^2}+\frac{1}{e'^2}=1$.

e) Prove that the centre of the circles $x^2+y^2=1$, $x^2+y^2+6x-2y-1=0$ and $x^2+y^2+12x+4y-1=0$ lie on a straight line.

f) Find the equation of the straight line $x\cos\alpha+y\sin\alpha=p$ when the axes are turned through an angle α without any change of origin.

g) Find α the asymptotes of the curve $xy-3x-4y=0$.

h) If $I_n=\int_0^{\frac{\pi}{2}} x^n \sin x dx, (n\geq 1)$, show that $I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$.

3. Answer any **two** questions: 5×2=10

a) Obtain the reduction formula for $\int \sec^n x dx$ and find the value of $\int \sec^6 x dx$.

b) i) Find the condition that one of the straight lines given by $ax^2+2hxy+by^2=0$ may coincide with one of the straight lines given by $a'x^2+2h'xy+b'y^2=0$.

ii) Show that x = y is one of the bisectors of the angle between the pair of straight lines $2x^2-7xy+2y^2=0$. 3+2

c) If $y = \cos\left(m \sin^{-1} x\right)$ show that

i) $\left(1-x^2\right)y_2 - xy_1 + m^2 y = 0$

ii) $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(n^2 - m^2\right)y_n = 0$

4. Answer any **one** question: 10×1=10

a) i) Find the volume of the solid generated by revolving the cardioide $r = a(1-\cos\theta)$ about the initial line.

ii) Find the envelope of the curve $y = mx + \frac{a}{m}$, $m(\neq 0)$ be a parameter.

iii) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant. 4+3+3

b) i) Reduce the equation $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$ to its canonical form and identify the conic.

ii) Show that the locus of the poles of tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$, is also a parabola. 6+4

c) i) Evaluate the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$ about the line y = 0.

ii) If PSP′ be the focal chord of a conic, show that $\frac{1}{SP} + \frac{1}{SP'} = \frac{2}{l}$, where l is the semi-latus rectum.

iii) Find the asymptotes of $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$.

3+3+4

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