U.G. 1st Semester Examination - 2021
PHYSICS

Course Code : BPHSCCHC 101

Course Title : Mathematical Physics I

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** of the following questions $1\times10=10$

a) State the condition of differentiability of a function.

b) Using Maclaurin's series expand $\sin x$ upto three terms.

c) If $\vec{P}$ and $\vec{Q}$ are each irrotational, prove the $\vec{P}\times\vec{Q}$ is solenoidal.

d) Find the order and degree of the differential equation:

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/2}+xy=0.$$

e) Give an example where Lagrange's undetermined multiplier is used.

f) State Gauss's divergence theorem.

g) Find the value of p for which the following three vectors are coplanar:

$\vec{a}=3\hat{i}+2\hat{j}+\hat{k}, \vec{b}=3\hat{i}+4\hat{j}+5\hat{k}, \vec{c}=\hat{i}+\hat{j}-p\hat{k}$.

h) Find the directional derivative of $f(x,y,z)=xy^2+yz^3$ at the point (2,-1,1) in the direction of the vector $\hat{i}+2\hat{j}-2\hat{k}$.

i) Evaluate : $\vec{\nabla}^2 \ln r$.

j) Find the *Wronskian* of solution, of a differential equation, which is given by $y=c_1\cos 2x+c_2\sin 2x$.

k) Using Gauss's divergence theorem prove that $\oint_s \vec{r}.\vec{ds}=3V$, where the surface *S* encloses the volume *V*.

l) What do you mean by *scale factors* in curvilinear coordinate system?

m) Answer whether the work done in a conservative force field is an exact or inexact differential.

n) What is the relation between *variance* and *standard deviation* of a statistical data?

o) When a differential equation is called linear?

2. Answer any **five** questions : 2×5=10

a) Find out the integrating factor and hence solve

$(x^2+y^2+x)dx+xydy=0$

b) If $\vec{\nabla}.\vec{E}=0, \vec{\nabla}.\vec{H}=0, \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{H}}{\partial t}, \vec{\nabla}\times\vec{H}=\frac{\partial\vec{E}}{\partial t}$,

show that $\vec{E}$ and $\vec{H}$ satisfy the wave equation

$\nabla^2\vec{u}=\frac{\partial^2\vec{u}}{\partial t^2}$.

c) Using Euler's theorem show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u\log u$,

where $\log u=(x^3+y^3)/(3x+4y)$.

d) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\hat{i}+\sin\omega t\hat{j}$, where ω is a constant. Show that the acceleration $\vec{a}$ is directed toward the origin.

e) What is the chance that a leap year selected at random will contain 53 fridays?

f) If $\vec{\upsilon}=\vec{\omega}\times\vec{r}$, prove $\vec{\omega}=\frac{1}{2}\text{curl}\,\vec{\upsilon}$ where $\vec{w}$ is a constant vector.

g) Show that the *Jacobian* of transformation from cartesian to spherical polar-coordinates

$$J\left(\frac{x,y,z}{r,\theta,\phi}\right)=r^2\sin\theta.$$

h) Evaluate the value of the integral:

$$\int_{-3}^{3}\delta(x-\pi)\sin x\,dx.$$

3. Answer any **two** questions : 5×2=10

a) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$.

b) A body of *m*, falling from rest is subject to the force of gravity and an air resistance proportional to the square of the velocity $(\text{i.e.,} kv^2)$. If it falls through a distance x and possesses a velocity v at that instant, then prove that

$$\frac{2kx}{m}=\log\frac{a^2}{a^2-v^2}, \text{ where } mg=ka^2.$$

c) Find the expressions of divergence and curl operator in spherical polar coordinates.

$2\frac{1}{2}+2\frac{1}{2}=5$