U.G. 1st Semester Examination - 2021

MATHEMATICS

Course Code : BMTMCCHT101

Course Title : Calculus & Analytical Geometry (2D)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions from the following: $1\times10=10$

a) If $f(x)=\cos nx$, then write down the value of $f^{(n)}\left(\frac{\pi}{2}\right)$.

b) What is the value of the integral $\int_{-2021}^{2021} \sin^{2021}x\,\cos^{2021}x\,dx$?

c) Write down the angle between the pair of straight lines given by $x^2+2xy\sec\theta+y^2=0$.

d) What is the degree of the honogeneous function $f(x,y)=\dfrac{x^{\frac{2}{3}}+y^{\frac{2}{3}}+x^{\frac{1}{3}}y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}$?

e) What is the necessary and sufficient conditions for $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a pair of straight lines?

f) Transform the equation $\left(x^2+y^2\right)^2=a^2\left(x^2-y^2\right)$ to polar form.

g) Obtain the value of $\int_0^{\frac{\pi}{2}}\sin^{10}x\,dx$.

h) What are the vertical asymptotes of the curve $x^2y^2=x^2+y^2$?

i) What is the condition that the straight line $lx+my+n=0$ may be a tangent to the parabola $y^2=4ax$?

j) Evaluate $\lim_{x\to0}\dfrac{x^{2022}}{e^x}$.

k) If $x=r\cos\theta$ and $y=r\sin\theta$, find $\dfrac{\partial\theta}{\partial x}$.

l) If $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$, then find $\frac{d^2y}{dx^2}$.

m) Find the value of c, for which the equation $x^2+y^2+2x+4y+c=0$ represents a pair of straight lines.

n) Determine the nature of the conic $r(4-5\cos\theta)=1$.

o) Find the differential of arc length for the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

2. Answer any **five** questions: 2×5=10

a) If u = x, v = x+y and w = x+y+z, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=1$.

b) If $u=\sqrt{xy}\,f\left(\frac{y}{x}\right)$, show that $xu_x+yu_y=u$.

c) Find the pedal equation of the curve $2r=1+\cos\theta$.

d) If $xy=x^n\log x$, show that $xy_n=(n-1)!$.

e) Determine the points of inflexion of the curve $y=\sin x$.

f) Show that the sum of the intercepts of any tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{c}$, is a constant.

g) Show that the curve $y^3+3ax^2+x^3=0$ is everywhere concave to the x-axis.

h) Find the equation of the tangents to the circle $x^2+y^2+8x+10y-4=0$ which are parallel to the straight line $x+2y+3=0$.

3. Answer any **two** questions: 5×2=10

a) Show that the volume of revolution generated by the region enclosed by $y=\sqrt{x}$ and the lines y = 1, x = 4 about x-axis is $\frac{9\pi}{2}$. 5

b) Reduce the equation $3(x^2+y^2)+2xy=4\sqrt{2}(x+y)$ to its canonical form and determine the nature of the conic. Find also the eccentricity of the conic and the equations of the axes. 5

c) Chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touch the circle $x^2+y^2=d^2$. Find the locus of their poles. 5

4. Answer any **one** question: 10×1=10

a) i) If f be a homogeneous function of x, y, z of degree n and $f = f(u, v, w)$ also where, $u = f_x$, $v = f_y$ and $w = f_z$ are differential, show that

$$uf_u + vf_v + wf_w = \frac{n}{n-1} f.$$ 5

ii) If $f(x) = e^{-x} D^n (e^x x^n)$, where $D \equiv \frac{d}{dx}$, then show that

$$x f''(x) + (x+1) f'(x) - nf(x) = 0.$$ 5

b) i) If $I_n = \int (\sin x + \cos x)^n dx$, then show that

$$n\, I_n = 2(n-1) I_{n-2} - (\sin x + \cos x)^{n-2}.$$

5

ii) If f is a function of x, y and $x = r\cos\alpha - \theta \sin\alpha$, $y = r \sin\alpha - \theta\cos\alpha$ then show that $f_{xx} + f_{yy} = f_{rr} + f_{\theta\theta}$

and $f_{rr} f_{\theta\theta} - f_{r\theta}^2 = f_{xx} f_{yy} - f_{xy}^2$. 2+3

c) i) If the normal to the rectangular hyperbola $xy = c^2$ at $\left(ct_1, \frac{c}{t_1}\right)$ meets the curve at $\left(ct_2, \frac{c}{t_2}\right)$, then show that $t_1^3 t_2 = -1$.

ii) If one of the straight line of $ax^2 + 2hxy + by^2 = 0$ coincides with one of the straight line of $a'x^2 + 2h'xy + b'y^2 = 0$ and the remaining two straight lines are at right angle, then show that $h\left(\frac{1}{b} - \frac{1}{a}\right) = h'\left(\frac{1}{b'} - \frac{1}{a'}\right)$.

6+4

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