U.G. 3rd Semester Examination - 2021

MATHEMATICS

Course Code : BMTMGEHT10

Course Title : Basics of Higher Mathematics-1

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) If $z = 1 - i$, find arg z.

b) If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4+1=0$, find the value of $(\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)$.

c) Determine the number of negative real roots of the equation $x^3+x+1=0$

d) Write down the n–th derivative of the function $\log_e(ax+b)$, $ax+b>0$.

e) If $z=\sin^{-1}\left(\frac{y}{x}\right)+\log\left(1+x^2\right)$, find $\frac{\partial z}{\partial x}$.

f) Examine whether the vector field defined by $r = xi + yj + zk$ is solenoidal.

g) If $f = xy+yz+zx$, find ∇f at the point (1, 1, 1).

h) Determine whether the equations x+y=10 and 2x+2y=20 are consistent or not.

i) Evaluate the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 5 & 1 \\ 2 & 3 & 2 \end{pmatrix}$.

j) Find the condition that the system of linear equation AX=b is called homogeneous.

k) For what value of α, the system of equations $3x+\alpha y=4$ and $6x-10y=8$ has infinite solutions.

l) State Euler's theorem on homogeneous function of two variables.

m) Prove that an invertible matrix has an unique inverse.

n) Establish the necessary and sufficient condition for a vector $\vec{r}=f(t)$ to have a constant direction.

o) Show that the equation $x^3+x^2-5x-1=0$ has two negative roots lying in (–1, 0) and (–3, –2).

2. Answer any **five** questions: 2×5=10

a) Verify Euler's theorem for the function $u = x^2 - xy + y^2$.

b) Find the Eigen values of the matrix $\begin{pmatrix} 0 & 8 \\ -2 & 0 \end{pmatrix}$.

c) Show that the vector field defined by $F = yzi + zxj + xyk$ is irrotational.

d) If $r = \sin t\, i + \cos t\, j + 2tk$, show that $\left|\frac{d^2r}{dt^2}\right| = 1$.

e) If $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ then show that $\frac{d^2y}{dx^2} = \frac{1}{4a}\sec^4\frac{\theta}{2}$.

f) Show that A^TA is Symmetric when A is m×n matrix.

g) Prove that $\sqrt{i} + \sqrt{-i} = \sqrt{2}$

h) State Leibnitz's theorem on the derivative of the product of two derivable functions of x.

3. Answer any **two** questions: 5×2=10

a) If $v = f(u)$ where $u = u(x, y)$ is a homogeneous function of degree *n*, show that

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = nu\frac{\partial v}{\partial u}.$$ 5

b) i) If α, β, γ be the roots of the equation $x^3 + ax^2 + bx + c = 0 (a \neq 0)$, find the equation whose roots are $\beta\gamma, \gamma a, \alpha\beta$.

ii) State De Moivre's theorem. 3+2

c) If $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ then find A^2 and show that $A^2 = A^{-1}$. 3+2

4. Answer any **one** question: 10×1=10

a) i) If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$

Find also the value of $(y_n)_0$.

ii) If *r* be the distance of any point P(x, y, z) from the origin, if *r* be the position vector of *P* relative to origin, show that $\nabla r^m = mr^{m-2}r$. 5+5

b) i) Solve the following system of linear equations by Cramer's rule.

$x + y = 2,\ y + z = 5,\ z + x = 3$.

ii) Prove that the sum of 99th powers of all the roots of $x^7 - 1 = 0$ is zero.

5+5

c) i) Solve the equation $x^3 - 6x - 9 = 0$ by using Cardon's method.

ii) Applying Divergence theorem evaluate $\iint_S F.nds$ where $\vec{F} = 3xz\vec{i} + y^2\vec{j} - 3yz\vec{k}$ and S is the surface of the cube bounded by x=0, y=0, z=0, x=1, y=1, z=1.

5+5

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