U.G. 5th Semester Examination - 2021

MATHEMATICS

Course Code: BMTMGERT10

Course Title: Basic of Higher Mathematics-I

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) If z=1+i, find arg z.

b) Under which condition $x^3+px^2+qx+r=0$ is a reciprocal equation of second kind?

c) If $z=\tan^{-1}(xy)$, find $\frac{\partial z}{\partial x}$.

d) What is the degree of the homogeneous function $u=\sin\left(\frac{y}{x}\right)$?

e) Find the divergence of the vector field defined by $r=xi+yi+zk$.

f) Examine whether the homogeneous system of equations $2x-3y=0$ and $3x+4y=0$ has a non trivial solution.

g) Express (–1–i) in polar form.

h) Find the cube roots of (–1).

i) Find the remainder when $x^5+5x^4+x^3+5x^2+2x+11$ is divided by x+5.

j) If $y=\sin(ax+b)$, find the n^{th} derivative y_n, where a, b are constants.

k) Find the remainder term R_n in the n^{th} order Taylor polynomial centered at a=2 for the function $f(x)=e^{-2x}$.

l) If $u=x\sin^{-1}\left(\frac{y}{x}\right)+y\tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ at (1, 1).

m) Give an example of matrix which has no real eigen value(s).

n) State Cayley-Hamilton theorem.

o) What is the number of solutions of the non-homogeneous system of linear equations

$$x-4y+6z=3$$
$$2x-y+3z=1$$

2. Answer any **five** questions: 2×5=10

a) If $y=x^2e^{5x}$ find y_6.

b) Verify Euler's theorem for the function $u = x^2+xy$.

c) Find the rank of the matrix $\begin{pmatrix} 1 & 3 & -2 \\ 2 & 6 & -4 \end{pmatrix}$.

d) Show that the vector field defined by F=yzi+zxj+xyk is irrotational.

e) Find the complex number z such that exp z = –1.

f) Apply Descartes' rule of signs to examine the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

g) Find the charcteristics equation of the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

h) A particle moves along a curve $x=e^{-t}, y=2\cos 3t, z=2\sin 3t$, where t is the time. Determine its velocity at $t=\pi$.

3. Answer any **two** questions: 5×2=10

a) i) If α, β, γ be the roots of the equation $x^3+ax^2+bx+c=0(a,c\neq 0)$, find the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

ii) State De-Moivre's theorem. 3+2=5

b) i) If $u=xyf\left(\frac{y}{x}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u$ by Euler's theorem.

ii) Prove that the Lagrange's remainder after n terms in expansion of e^{ax} in powers of x is

$$\frac{\left(a^2+b^2\right)^{\frac{n}{2}}}{n!}x^n e^{a\theta x}\cos\left(b\theta x+n\tan^{-1}\frac{b}{a}\right), 0<\theta<1.$$

2+3=5

c) i) Show that the equation $\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$ represents the rectangular hyperbola $x^2-y^2=xy$.

ii) Solve the reciprocal equation $x^4+10x^3+26x^2+10x+1=0$.

2+3=5

4. Answer any **one** question: 10×1=10

a) i) If $y=e^{a\sin^{-1}x}$ prove that

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+a^2\right)y_n=0.$$

ii) If r be the distance of any point *P*(x, y, z) from the origin, if *r* be the position vector of *P* relative to origin, show that $\nabla r^m = m r^{m-2} r$ · 5+5=10

b) i) Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

ii) Vertify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$.

iii) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$.

4+3+3=10

c) i) If $y = \sin^{-1} x$ then show that $(1-x^2)y_2 - xy_1 = 0$ and $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.

ii) Find the directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at the point (2, 1, 3) in the direction of the vector $\hat{i} - 2\hat{k}$ ·

iii) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2, –1).

(1+3)+3+3=10

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