U.G. 5th Semester Examination - 2021

PHYSICS

Course Code : BPHSDSHC1 [DSE 1]

Course Title : Advanced Mathematical Physics

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

   a) What do you mean by dimension of a vector space?

   b) Define inner product space.

   c) If the set of vector (0, –1, 2), (0, 1, 6), (1, 2, x) are linearly independent, then find the value of x.

   d) Determine whether or not the matrices $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ form a basis.

   e) Write down the transformation rule for a 2nd rank convariant tersor.

   f) What is similarity transformation?

   g) A square matrix 'A' is diagonalised to 'D' by the transformation $P^{-1}AP=D$. Prove that if A be a hermitian matrix, p is unitary.

   h) If $cf(x)=f^2(x)$ then show that c is not a linear operator.

   i) What do you mean by unitary operator?

   j) Write down the number of independent component of a symmetric tensor A_{ij} with indices i, j =1, 2, 3.

   k) $\vec{c}_1, \vec{c}_2$ and $\vec{c}_3$ form a basis set in a three dimensional vector space. Write down its dual basis vectors.

   l) What are symmetric and antisymmetric tensors?

   m) Obtain the metric tensor for two dimensional plane in polar coordinates.

   n) If A and B are linear operator then prove that $(AB)^{-1} = B^{-1}A^{-1}$.

   o) What do you mean by Geodesic path?

2. Answer any **five** questions: 2×5=10

   a) Show that the two vector $u=(1+i, 2-i, -3)$ and $v=(3i, -1+2i, 2+i)$ satisfy the Schwarz inequality.

b) Find the eigen values of the hermitian matrix

$$B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

c) Determine whether or not the set of vectors $\{(1,1,2),(1,2,5),(5,3,4)\}$ is a basis of the vector space $\mathbb{R}^3$.

d) If $\hat{A}$ and $\hat{B}$ are the linear operators in the vector space V. show that their product $\hat{A}\hat{B}$ is also a linear operator.

e) A linear operator $F:\mathbb{R}^2 - \mathbb{R}^2$ is defined by F(x, y)=(2x+3y, 4x–5y) find the matrix representation of F relative to the basis S = {(1, 2), (2, 5)}.

f) Show that any tensor of rank 2 can be expressed as the sum of a symmetric and an anti symmetric tensor of rank-2.

g) In special relativity the space time interval is given by $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$
Write down its metric tensor. Whether this space time is flat or curved?

h) If x, y, z are linearly independent vectors, determine whether (x+y), (y+z), (z+x) are lineraly independent or dependent.

3. Answer any **two** questions: 5×2=10

a) Obtain a set of four orthonormal vectors by the Schmidt's method from the vectors
$u_1 = (1,1,01),\ u_2 = (2,0,0,1)$
$u_3 = (0,2,3,-2),\ u_4 = (1,1,1,-5)$

b) Find the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis (1, 0, 0), (0, 1, 0), (0, 0, 1) to (1, 1,1), (0, 1,–1) and (1, 2, 0) respectively. Find the image of (2, 2, –2) and (3, 3, –3) under this transformation. Does this reflect one to one correspondence?

2+2+1

c) i) Examine whether $A = \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}$ is a tensor or not.

ii) Separate the tensor $\begin{pmatrix} -xy & x^2 \\ -y^2 & xy \end{pmatrix}$ into symmetric and antisymmetric tensor.

3+2

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