# U.G. 3rd Semester Examination - 2021

# MATHEMATICS

**Course Code : BMTMCCRT301**

**Course Title : Geometry-3D and Vector Analysis**

Full Marks : 40 Time : 2 Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Notations and Symbols have their usual meanings.*

1. Answer any **ten** questions: 1×10=10

a) Find the co-ordinates of the centre of the sphere $(x-2)(x+1)+(y-1)(y-3)+(z+1)(z-2)=0$.

b) Find the equation of the tangent plane to the sphere $3(x^2+y^2+z^2)=4$ at the point (–1, –2, 3).

c) Write down the general equation to the cone which passes through the axes.

d) Show that $\vec{\nabla}.\vec{r}=3$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

e) Define a ruled surface.

f) Write the equation of the enveloping cone of $ax^2+by^2=2cz$ with vertex at $(\alpha, \beta, \gamma)$.

g) Define reciprocal cones.

h) State Stokes' theorem.

i) Find the radius of the circle $x^2+y^2+z^2=25,\ x+2y+2z+9=0$.

j) Find the surface of revolution of the straight line $y=z\tan\theta, x=0$ about the z-axis.

k) If $\vec{a}(u)=u^2\hat{i}+(u-1)\hat{j}-4\hat{k}$, then find $\int \vec{a}(u)du$.

l) Show that the curl of a gradient is zero.

m) If $\vec{r}=e^{xy}\hat{i}+(2x-y)\hat{j}+y\sin x\hat{k}$, then find $\frac{\partial\vec{r}}{\partial x}$ and $\frac{\partial^2\vec{r}}{\partial x^2}$.

n) What type of surface is represented by the equation $3x^2-2y^2=6z$.

o) Show that the vector $\{(\vec{\alpha}.\vec{\gamma})\vec{\beta}-(\vec{\alpha}.\vec{\beta})\vec{\gamma}\}$ is perpendicular to the vector $\vec{\alpha}$.

2. Answer any **five** questions: $2\times5=10$

a) Find the equation of the right circular cylinder whose axis is the x-axis and radius 1.

b) State Serret-Frenet formula for a space curve.

c) Determine the value of the constant d such that the vectors $\left(2\hat{i}-\hat{j}+\hat{k}\right), \left(\hat{i}+2\hat{j}+d\hat{k}\right)$ and $\left(3\hat{i}-4\hat{j}+5\hat{k}\right)$ are coplanar.

d) Evaluate $\int_1^2\left(\vec{r}\times\frac{d^2\vec{r}}{dt^2}\right)dt$ where $\vec{r}=2t^2\hat{i}+t\hat{j}-3t^2\hat{k}$.

e) Find the equation of the normal of $2x^2+3y^2=4z$ at the point (2, 2, 5).

f) Show that the straight line x–1=y–2=z+1 lies entirely on the surface $z^2-xy+2x+y+2z-1=0$.

g) Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F}=x^2y^2\hat{i}+y\hat{j}$ and the curve C is $y^2=4x$ in the xy-plane from (0, 0) to (4, 4).

h) If $\vec{\alpha}=\hat{i}+\hat{j}-6\hat{k}, \vec{\beta}=\hat{i}-3\hat{j}+4\hat{k}, \vec{\gamma}=2\hat{i}-5\hat{j}+3\hat{k}$ then find $\vec{\alpha}.\left(\vec{\beta}\times\vec{\gamma}\right)$.



3. Answer any **two** questions: $5\times2=10$

a) The section of a cone whose guiding curve is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane x=0 is a rectangular hyperbola. Show that the locus of the vertex is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$. 5

b) Use divergence theorem to evaluate $\iint_S\left(x^3dydz+x^2ydzdx+x^2zdxdy\right)$ where S is the closed surface bounded by the planes z = 0, z = b and the cylinder $x^2 + y^2 = a^2$. 5

c) Find the equation of the sphere of which the circle $xy+yz+zx=0, x+y+z=3$ is a great circle. 5

4. Answer any **one** question: $10\times1=10$

a) i) Reduce the equation: $x^2+y^2+z^2-2xy-2yz+2zx+x-4y+z+1=0$ to its canonical form and determine the type of quadric representaed by it.

ii) Prove that the five normals from a given point to a paraboloid lie on a cone. 6+4

b) i) Find the locus of the points of intersection of the perpendicular generators of the hyperbolic paraboloid

$$\frac{y^2}{b^2} - \frac{z^2}{c^2} = 2x. \quad 5$$

ii) Find the equation of the sphere touching the three co-ordinate planes. 3

iii) Find the equation of the surface of revolution whose generatrix is $x^2 - 4z = 0$, $y = 0$ and the axis is the z=axis. 2

c) i) Use divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\, ds$, where $\vec{F} = 3xz\,\hat{i} + y^2\hat{j} - 3yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$.

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ii) Verify Green's theorem in a plane for $\oint_c \{(x^2 + xy)dx + xdy\}$, where C is the curve enclosing the region bounded by $y = x^2$ and $y = x$. 5

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