U.G. 3rd Semester Examination - 2021

PHYSICS

Course Code : BPHSCCHC301

Course Title : Mathematical Physics II

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The symbols have their usual meaning.

1. Answer any **ten** questions: 1×10=10

a) Explain periodic function with an example.

b) State Dirichlet's conditions for a function to be expanded as a Fourier series.

c) Write down the relation between Beta and Gamma functions.

d) Plot $J_0(x)$ and $J_1(x)$ *vs* x.

e) Define regular and irregular singular points of a differential equation.

f) From the generating function of Legendre polynomials, find the value of $P_n(-1)$.

g) Write down one dimensional heat equation.

[Turn Over]

h) Write down the general differential equation of *n*-th order. When it is called homogeneous?

i) Show that $\beta(m,n)=\beta(n,m)$.

j) State the type of the following differential equation:

$$9\frac{\partial^2 u}{\partial x^2}+6\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}=3x+4y+1$$

k) What do you mean by action?

l) Define cyclic coordinates.

m) State Euler's equation.

n) Express $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta$ in gamma function.

o) State Parseval's formula.

2. Answer any **five** questions: 2×5=10

a) Evaluate the value of $\Gamma\left(\frac{1}{2}\right)$.

b) Using Euler's equation find the extremals of the functional $\int_{x_0}^{x^1}\left(\frac{y'^2}{x^3}\right)dx$. y' represent first derivative.

c) Solve : $\frac{d^2y}{dx^2}+4y=\sin 2x$

d) Find the Hamilton's canonical equations for a simple pendulum.

e) Express $5x^3 - x + 2$ in terms of Legendre's polynomial.

f) Show that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p,q)}{p+q}$.

g) Obtain the value of $P_0(x)$ and $P_1(x)$ using Rodrigue's formula.

h) Using d'Alembert's method, find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = k(\sin x - \sin 2x)$.

3. Answer any **two** questions: 5×2=10

a) Solve in series the equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$$

b) Show that

i) $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$

ii) $J_n'(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$ $2\frac{1}{2}+2\frac{1}{2}$

c) Obtain Fourier series for the function f(x) given by

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \le x \le 0$$

$$= 1 - \frac{2x}{\pi}, 0 \le x \le \pi.$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$.