U.G. 5th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMDSRT-1 & 2 (DSE 1 & 2)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

This question papers contains both DSE 1 & 2. Students are thereby instructed to answer DSE paper out of these two (DSE 1 & DSE 2) as he/she opted for.

Course Title : Linear Programming

Course Code : BMTMDSRT-1 (DSE 1)

1. Answer any **ten** questions: 1×10=10

   a) Give an example of a convex set.

   b) Write down the net evaluation assoiciated with j-th vector.

   c) What is the difference between slack and surplus variable?

   d) Is hyperplane a convex set?

   e) Define saddle point of a rectangular game.

   f) Why is the name Linear Programming Problem?

   g) Do the vectors (1, 2, 3), (4, 5, 6) and (3, 6, 9) form a basis for E^3? Justify.

   h) Define basic solution of a system of linear equations.

   i) Define an extreme point of a convex set.

   j) State fundamental theorem of LPP.

   k) Find the dual of the LPP:

   Maximize $Z = 4x_1 + 3x_2$

   Subject to $x_1 + x_2 \le 5$
   $2x_1 - 3x_2 \le 2$
   $x_1, x_2 \ge 0$

   l) Define an unbalanced transportation problem.

   m) Express $x_1 - 5x_2 + 3x_3 = 7$ into two inequations.

   n) When a game is called a two person zero sum game?

o) Reduce one row of the following game problem mentioning the law:

Player B

Player A			
	1	7	2
	6	2	7
	5	1	6

2. Answer any **five** questions: 2×5=10

a) Examine whether the vectors (1, 1, 0), (0, 1, 1) and (1, 0, 1) form a basis for E^3 or not.

b) What do you mean by degenerate solution?

c) Prove that the intersection of two convex sets is also a convex set.

d) Reduce the following LPP to its standard form:

Maximize $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

e) Write down all the conditions of a set of cells in a tranportation table to form a loop.

f) Show that the following game problem is strictly determinable and fair:

Player B

Player A		
	0	2
	−1	4

g) Determine the position of the points (1, 2, 3) and (0, 1, –3) with respect to the hyperplane $x + 2y + 3z = 9$.

h) Sketch graphically the region in the first quadrant satisfying the following inequalities:

$$x_1 + x_2 \le 3,\ x_1 + 2x_2 \le 4$$

3. Answer any **two** questions: 5×2=10

a) Use simplex method to obtain the inverse of the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$.

b) Solve the following LPP by simplex method:

Maximize $Z = 7x_1 + 5x_2$

Subject to $x_1 + 2x_2 \le 6$

$4x_1 + 3x_2 \le 12$

$x_1, x_2 \ge 0$

c) Solve the following assignment problem:

	I	II	III	IV
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

4. Answer any **one** question: 10×1=10

a) i) Is $x_{13} = 50, x_{14} = 20, x_{21} = 55, x_{31} = 30, x_{32} = 35, x_{33} = 25$ an optimal solution of the following transportation problem?

				a_i
6	1	9	3	70
11	5	2	8	55
10	12	4	7	90
85	35	50	45	b_j

If not, modify it to obtain a beter feasible solution. Find the solution of the modified problem.

ii) Convert the following peoblem to a dual problem:

Min $Z = 3x_1 - 2x_2 + 4x_3$

S.T. $3x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \leq 4$

$7x_1 - 2x_2 - x_3 = 10$

$x_1, x_2 \geq 0$, x_3 is unrestricted in sign.

6+4

b) i) Prove that $x_1 = 2, x_2 = 1, x_3 = 3$ is a feasible solution of the system:

$4x_1 + 2x_2 - 3x_3 = 1$

$6x_1 + 4x_2 - 5x_3 = 1$

Reduce it to a basic feasible solution.

ii) Using two phase method, solve

Max $Z = 2x_1 + x_2 - x_3$

S.T. $4x_1 + 6x_2 - 3x_3 \leq 8$

$3x_1 - 6x_2 - 4x_3 \leq 1$

$2x_1 - 3x_2 - 5x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

(1+2)+7

c) i) Solve the following LPP by Charne's Big M mehod:

Minimize $Z = 3x_1 + 5x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60$

$x_1, x_2 \geq 0$

ii) Prove that a necessary and sufficient condition for the existence of a feasible solution to a transportation problem is that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

6+4

Course Title : Mechanics-I

Course Code : BMTMDSRT-2 (DSE 2)

1. Answer any **ten** questions: $1\times10=10$

a) State Newton's laws of motion.

b) State D'Alembert's principle.

c) Obtain the formula for the time rate of change of angular momentum about a fixed point.

d) Give an example of effective force and impressed force.

e) Is a coordinate frame rigidly attached to the rotating earth's surface an inertial frame? Justify.

f) Define centre of mass of a system of particles.

g) What do you mean by potential energy of a system?

h) What do you mean by torque of a force about a point?

i) State law of inertia.

j) What do you mean by viscocity of a fluid?

k) Define conservative force field.

l) What are the physical significance of the eigenvalues and eigenvectors of the inertia matrix?

m) Define degree of freedom of an object.

n) What are principal axes? What is the form of inertia matrix with respect to principal axes?

o) Give an example of non-inertial frame.

2. Answer any **five** questions: $2\times5=10$

a) Can the principle of virtual work be applied to bodies in motion? Explain.

b) Prove that, the moment of a force about the origin of a coordinate system is equal to the rate of change of angular momentum.

c) In a conservative field of force $\vec{F}$, write down the mathematical expression of potential function V. Explain why the absolute value of the potential energy is meaningless.

d) Can a rigid body be deformable?

e) State the principle of transmissibility of force acting on a rigid body.

f) If A and B are the moments and F be the product of inertias of a uniform lamina about two rectangular axes OX and OY. Show that principal moments are equal to $\frac{1}{2}\left[A+B\pm\sqrt{(A-B)^2+4F^2}\right]$.

g) Show that the distance between two points is invariant in two inertial frames.

h) Calculate the virtual work done by the reaction of a body sliding on a smooth surface.

3. Answer any **two** questions: $5\times2=10$

a) i) What do you understand by a constraint in a dynamical system? Give examples.

ii) Give the concept of 'absolute time' and 'absolute length'. (2+1)+(1+1)=5

b) Find the time of a complete oscillation of compound pendulum. 5

c) A particle moving in a straight line is acted on by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x. Prove that the time taken is $\frac{3(u+v)x}{2(u^2+uv+v^2)}$. 5

4. Answer any **one** question: $10\times1=10$

a) i) State and prove the theorem of principal axes. 1+5

ii) Show that, at the centre of a quadrant of an ellipse the principal axes in its plane are inclined at an angle $\frac{1}{2}\tan^{-1}\left(\frac{4}{\pi}\frac{ab}{a^2-b^2}\right)$. 4

b) i) A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding, Find the motion. 6

ii) A plank, of mass M, is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man, of mass M^1, starting from the upper end walks down the plank, so that it does not move; show that he gets to the other end in time $\sqrt{\frac{2M^1a}{(M+M^1)g\sin\alpha}}$ where a is the length of the plank. 4

c) A small insect moves along a uniform bar of mass equal to itself and of length 2a, the ends of which are constrained to remain on the circumference of a fixed circle whose radius is $\frac{2a}{\sqrt{3}}$. If the insect starts from the middle point of the bar and moves along the bar with relative velocity V, then show that the bar in time t will turn through an angle $\frac{1}{\sqrt{3}}\tan^{-1}\frac{Vt}{a}$. 10

———————