U.G. 5th Semester Examination - 2021

PHYSICS

Course Code : BPHSCCHC501

Course Title : Quantum Mechanics and Applications

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: $1\times10=10$

a) The time independent wave function of a particle in one-dimensional box is given as $\psi(x)=\frac{1}{\sqrt{L}}\sin\left(\frac{n\pi x}{L}\right)$. What is the dimension of the given wavefunction $\psi(x)$?

b) Write down the time independent Schrodinger equation of a free particle.

c) If $\phi_1(x)$ and $\phi_2(x)$ are two Eigen-states of a one-dimensional system, what is the general form of the wavefunction of the system?

d) $\psi(x)=e^{-x}$ is not a possible wavefunction. Is the statement correct? Please justify the answer.

e) $\psi(x)=Ae^{-x^2}$, $x\in\mathbb{R}$. Find the value of A, if $\psi(x)$ is normalized.

f) What is the physical condition for having discrete energy levels for a system?

g) $\psi(\vec{r})$ is the wave function of a particle. What is the dimension of $|\psi(\vec{r})|^2$?

h) Show that $\left[x,\frac{\partial^2}{\partial x^2}\right]=-2\frac{\partial}{\partial x}$.

i) If $\hat{A}$ and $\hat{B}$ be two Hermitian operator, then show that the commutator $[\hat{A},\hat{B}]$ is an anti-Hermitian operator.

j) Can the principal quantum number of an electron in hydrogen atom be zero?

k) Explain degeneracy of a state. Give example.

l) What is meant by space quantization?

m) What do you understand by fine structure splitting?

n) What is Paschen-Back effect?

o) Write down the relation between magnetic moment $\vec{\mu}$ and the orbital angular momentum $\vec{L}$ of an electron.

2. Answer any **five** questions: 2×5=10

a) Show that $\hat{x}$ and $\hat{p}_x$ operators do not commute. What is the physical meaning of the result?

b) Prove that $m\frac{d}{dt}\langle x\rangle = \langle p_x\rangle$

c) If $R_{nl}(r)$ is a radial wave function of the electron in hydrogen atom, what is the probability of existence of the electron between r and r + dr?

d) What is the value of ground state energy of an one-dimensional simple harmonic oscillator? Why this is not equal to zero?

e) A system has two energy eigenvalues ε_0 and $3\varepsilon_0$, and ψ_1 and ψ_2 are two corresponding normalized wave function. At an instant the system is in a state $\psi = \frac{1}{\sqrt{2}}\psi_1 + C\psi_2$. Find the value of C if ψ is normalized and also find out expectation value of energy.

f) The energy eigenvalue and the corresponding eigen function for a particle of mass m in one-dimensional potential $V(x)$ are $E = 0,\ \psi = \frac{A}{x^2 + a^2}$, $A = \text{constant}$. Find the expression for the potential V(x).

g) Evaluate the Lande's g-factor for $^2D_{3/2}$ state.

h) Explain gyromagnetic ratio and Bohr magneton.

3. Answer any **two** questions: 5×2=10

a) Find the state wave functions of a particle of mass m and confined in an one-dimensional box of lengths L. The particle is confined with infinite potentials at $x = -\frac{L}{2}$ and $x = +\frac{L}{2}$. Find the parity of the state functions.

b) For a one-dimensional simple harmonic oscillator the following operators are defined:

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} + \frac{i}{\sqrt{2m\omega\hbar}}\hat{p}$$

$$\widehat{a^{+}} = \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x} - \frac{i}{\sqrt{2m\omega\hbar}}\hat{p}$$

Show that $\hat{H} = \hbar\omega\left(\widehat{a^{+}}\,\hat{a} + \frac{1}{2}\right)$.

c) Write down Schrödinger equation for the electron of Hydrogen atom assuming the nucleus to be stationary. By separation of variable, obtain the radial equation. The normalized wave function of the ground state of the H-atom is given by $\psi(r)\frac{1}{\sqrt{\pi}\,a_0^{3/2}}e^{-r/a_0}$, a_0=Bohr-radius.

Find the distance from the nucleus at which the electron is most likely to be found.

1+2+2=5
