U.G. 2nd Semester Examination - 2021

MATHEMATICS

[PROGRAM]

Course Code : BMTMCCRT201

Course Title : Ordinary Differential Equations and Linear Algebra

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Write down the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}} + 2.\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^5$.

b) Define exact differential equation.

c) Find an integrating factor of the differential equation $(xdy - ydx) = 0$.

d) Find the value of *m* which makes the differential equation $(a^2 - mxy - y^2)dx - (x + y)^2dy = 0$ exact.

e) Form the differential equation whose primitive is $ax + by + c = 0$, *a,b,c* being parameters.

f) For what value of λ, $e^{\lambda x}$ will be trial solution of the equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$?

g) Define trajectories to a given family of curves.

h) Find the *Wronskian* of the functions $e^x, \cos x$.

i) Write down the standard basis of the space $\mathbb{R}^3$.

j) What is the linear span of an empty set of vectors?

k) Define column rank of a vector space.

l) When the set of vectors $\{\alpha, \beta, \gamma\}$ is called independent?

m) Solve the equation $2x + 3y + 5z = 0; (x, y, z) \in \mathbb{R}^3$.

n) Show that the set $V = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$ is not a subspace of $\mathbb{R}^3$.

o) Give an example of an infinite dimensional vector space.

2. Answer any **five** question. 2×5=10

a) Show that $\frac{1}{x^2y^2}$ is an integrating factor of the differential equation $4x^3y\,dx + (x^4 + y^4)\,dy = 0$.

b) Solve the differential equation $(x^2+1)^3\frac{dy}{dx} + 4x(x^2+1)^2 = 1$.

c) Find the general solution of the differential equation $y = px + \sin p$; where $p \equiv \frac{d}{dx}$.

d) Find the vatue of $\frac{1}{D^2-9}\{e^{3x}\}$; where $D \equiv \frac{d}{dx}$.

e) Write down the condition for integrability of Pfaffian differential equation.

f) Give the geometrical interpretation of the equation $\frac{dx}{P(x,y,z)} = \frac{dy}{Q(x,y,z)} = \frac{dz}{R(x,y,z)}$.

g) Examine whether or not $S = \{(x,y,z) \in \mathbb{R}^3 : x = 0\}$ is a subspace of $\mathbb{R}^3$.

h) Show that the vectors (1, 2, 1), (3, 0,–5) are linearly independent.

3. Answer any **two** questions: 5×2=10

a) Solve the equation $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{xyz-2x^2}$.

b) Find the orthogonal trajectories to the family of curves $r = a(1+\cos\theta)$, a being a parameter.

c) If U and W be two subspaces of a vector space V, show that the linear sum $U+W$ is also a subspace of V.

4. Answer any **one** question: 10×1=10

a) i) Find the general solution and the singular solution of the equation $y = px + \frac{a}{p}$.

ii) Solve the system of equations $x + 3y + z = 0,\ 2x - y + z = 0.$. 5+5

b) i) Prove that a linearly independent set of vectors in a finite dimensional vector space is either a basis of V, or it can be extended to a basis of V.

ii) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$. 5+5

c) i) Solve the simultaneous linear equations $\frac{dx}{dt} = -\omega y, \frac{dy}{dt} = \omega x$.

ii) Find the dimension of the subspace S of $\mathbb{R}^3$ defined by $S = \{(x,y,z) \in \mathbb{R}^3 : 2x + y - z = 0\}$. Also find a basis of S. 5+5

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