U.G. 2nd Semester Examination - 2021

BCA

[HONOURS]

Course Code : BBCAGEHT2

Course Title: Mathematics-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

 a) When does an infinite series is said to converge?

 b) Define odd function with example.

 c) Evaluate $\int_{-\pi/2}^{\pi/2} \cos x dx$

 d) Find $\frac{dy}{dx}$, if $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$

 e) State the Euler's theorem on homogeneous function of two variables.

 f) State Rolle's theorem.

 g) Evaluate : $\lim_{x\to 3} \frac{x^3 - 27}{x - 3}$

 h) Evaluate : $\lim_{x\to\infty}\left(x - \sqrt{x^2 + x}\right)$

 i) If $y = x^{\sin x}$ then find $\frac{dy}{dx}$.

 j) In the Mean Value Theorem $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$ if h=3 and $f(x) = \sqrt{x}$ find the value of θ.

 k) Evaluate : $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$

 l) If $u = x^y$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

 m) Find the integrating factor of the linear differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$.

 n) Write down the formula for the integration of a product of two functions.

 o) Define Cauchy Sequence.

2. Answer any **five** questions: 2×5=10

 a) Verify that $y = \frac{A}{x} + B$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} = 0$.

b) If $x = f(t), y = \phi(t)$, then

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\cdot\frac{d^2y}{dt^2} - \frac{dy}{dt}\cdot\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

c) Differentiate $x^{\sin^{-1}x}$ with respect to $\sin^{-1}x$.

d) Find the integrating factor of

$$\cos x\frac{dy}{dx} + y\sin x = 1$$

e) If $v = x^2 + y^2 + z^2$ show that $xv_x + yv_y + zv_z = 2v$.

f) Integrate $\int e^{\tan^{-1}x}\frac{1}{1+x^2}dx$

g) Evaluate $\int_{-1}^{1} xe^{tx}dx$

h) If $y = \left(x + \sqrt{1+x^2}\right)^m$, prove that

$$\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

3. Answer any **two** questions: 5×2=10

a) Use Cauchy criterion to show that the sequence $\{x_n\}$ defined by

$$x_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + (-1)^{n-1}.\frac{1}{n}$$ is convergent.

b) Show that $\int_0^\infty \frac{dx}{(x+1)(x+2)} = \log 2$

c) Prove that:

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2-1^2}} + \frac{1}{\sqrt{n^2-2^2}} + \frac{1}{\sqrt{n^2-3^2}} + \ldots + \frac{1}{\sqrt{n^2-(n-1)^2}}\right] = \frac{\Pi}{2}.$$

4. Answer any **one** question: 10×1=10

a) i) Obtain the general solution and singular solution of the equation

$$y = px + \sqrt{a^2p^2 + b^2}$$

ii) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$

iii) Solve : $\cos x\frac{dy}{dx} + y\sin x = 1$ 4+3+3

b) i) Show that the sequence $\sqrt{7}, \sqrt{7+\sqrt{7}}, \sqrt{7+\sqrt{7+\sqrt{7}}}$, ... converges to the positive root of the equation $t^2 - t - 7 = 0$.

ii) Evaluate

$$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots + \frac{n}{n^2+n^2}\right].$$

5+5

c) i) Prove that

$$\int_0^{\pi} \frac{x\,dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{\Pi^2}{2ab}, \ (a, b > 0).$$

ii) If $y = e^{a \sin_x^{-1}}$ then prove that

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + a^2) y_n = 0$$

iii) If $v = \sin^{-1}\left\{\frac{x^2 + y^2}{x + y}\right\}$, then show that

$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} = \tan v.$$

4+3+3
