U.G. 4th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMGEHT10A

Course Title : Basics of Higher Mathematics-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Find the order and degree of

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}=\frac{d^2y}{dx^2}.$$

b) Find the value of $\int_0^{\frac{\pi}{2}}\sin^7 x\,dx$.

c) Find a point on the conic $\frac{l}{r}=1-\cos\theta$ having the least radius vector.

d) Give an example of a non-central conic.

e) Write down the condition of exactness for the differential equation Mdx+Ndy=0.

f) Find the equation of the plane passing through (–2, 3, 10) and through the z-axis.

g) What is Bernoulli's equation?

h) Write the equation of the sphere having segment joining (1, 2, 1) and (3, 4, 5) as diameter.

i) What is the number of generators in an infinite cyclic group?

j) The set $\mathbb{Z}$ of integers is a group under the binary operation '$*$' defined by $m*n=m+n+1$; $m, n\in\mathbb{Z}$. What is the inverse of the element 5?

k) Find the area of the cardioid $r=a(1-\cos\theta)$.

l) Define intrinsic equation of a curve.

m) Obtain a particular integral of $\left(D^2+D-2\right)y=e^x$.

n) Find the distance of (5, 3, 4) from x-axis.

o) If $I_n=\int e^{-x}x^n\,dx$, establish the formula $I_n=e^{-x}x^n+nI_{n-1}$.

2. Answer any **five** questions: 2×5=10

a) If the origin be shifted to the point (0, 1) and the axes be rotated through an angle 45°, then find the transformed form of the equation $5x^2-2xy+5y^2+2x-10y-7=0$.

b) Find all cyclic subgroup of the group $(S, \bullet)$, where $S=\{1, i, -1, -i\}$.

c) Obtain the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as the great circle.

d) Define linear ordinary differential equation. Give an example.

e) Show that the condition that the bisectors of the angles between the straight lines $ax^2+2hxy+by^2=0$ and $a'x^2+2h'xy+b'y^2=0$ are the same is $h(a'-b')=h'(a-b)$.

f) Show that the ring of matrices $\left\{\begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b \in \mathbb{Z}\right\}$ contains divisors of zero and does not contain the unity.

g) Determine an integrating factor of the differential equation

$$y(1+xy)dx+x(1-xy)dy=0.$$

h) Show that the straight line $\frac{l}{r}=A\cos\theta+B\sin\theta$ touches the conic $\frac{l}{r}=1+e\cos\theta$ if $(A-e)^2+B^2=1$.

3. Answer any **two** questions: 5×2=10

a) i) Solve the differential equation

$$\frac{dy}{dx}=\sin(x+y)+\cos(x+y). \quad 3$$

ii) In a group $(G, \circ)$, a is an element of order 30. Find the order of a^{18}. 2

b) A variable plane has intercept on the coordinate axes, the sum of whose square is k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is

$$(x^2+y^2+z^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)=k^2.$$

c) Find the whole length of the loop of the curve $9ay^2=(x-2a)(x-5a)^2$.

4. Answer any **one** question: 10×1=10

a) i) Find a reduction formula for $I_{m,n} = \int_0^{\frac{\pi}{2}} \cos^m x \sin nx \, dx$, $m, n \in I^+$ and hence deduce that

$$I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \ldots + \frac{2^m}{m}\right].$$

ii) Reduce the equation $x^2-5xy+y^2+8x-20y+15=0$ to its canonical form and determine the type of the conic represented by it.

b) i) In a group $(G, \circ)$, the elements a and b commute and $O(a)$ and $O(b)$ are prime to each other. Show that $O(a \circ b) = O(a).O(b)$. 4

ii) Reduce the equation

$$\left(px^2 + y^2\right)(px + y) = (p+1)^2,$$

where $p \equiv \frac{dy}{dx}$, to Clairaut's form by using the substitution u=xy, v=x+y and hence find its complete primitive. 4

iii) Show that the area of the loop of the curve $r = a\theta\cos\theta$ lying in the first quadrant is $\frac{1}{96}\pi a^2\left(\pi^2 - 6\right)$. 2

c) i) Show that the equation to the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1$, x=0 and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1$, y=0 is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$ and if 2d is the shortest distance, prove that $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.

ii) Solve $\left(D^2 - 1\right)y = 50xe^{2x}\cos x$.

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