U.G. 4th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMCCRT 401

Course Title : Partial Differential Equation, Laplace Transform & Tensor Analysis

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Give an example of a first degree partial differential equation.

b) Obtain a P.D.E. by eliminating a and b from $az+b=a^2x+y$.

c) Find the order of the partial differential equation $\left(\frac{dz}{dx}\right)^3+\frac{dz}{dy}=0$.

d) Find L{f(t)}, where f(t)=2 and L{f(t)} denote the Laplace transform of the function f(t).

e) What is complete integral of p+q=1?

f) Find $L\{t^2-3t+2\}$.

g) State the sufficient condition for existence of a Laplace transform.

h) State convolution theorem for Laplace transform.

i) Define free index.

j) Define the formula for the angle between two non-null contravariant vectors A^i and B^j.

k) Define Christoffel symbols of second kind $\left\{\begin{matrix} p \\ i\,j \end{matrix}\right\}$.

l) Write down the dummy index (or indices) in the term $a_{ix}x^i$.

m) Write down the order and degree of the P.D.E. $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial t^2}=0$.

n) Write down the auxiliary equation of the Lagrange's equation $P\frac{\partial z}{\partial x}+Q\frac{\partial z}{\partial y}=R$, where P, Q, R are functions of x, y, z.

o) What is the value of $L^{-1}\left\{\frac{1}{p-a}\right\}$, where p>a.

2. Answer any **five** questions: 2×5=10

a) Form a partial differential equation by elimination of ϕ from

$$lx + my + nz = \phi\left(x^2 + y^2 + z^2\right).$$

b) Find the PDE of all planes which have equal x and y intercepts.

c) Solve: $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z$

d) Show that,

$$L\{a_1F_1(t) + a_2F_2(t)\} = a_1L_1\{F_1(t)\} + a_2L_2\{F_2(t)\}$$

where a_1, a_2 are constants.

e) Find $L^{-1}\left(\frac{4}{P-2}\right)$.

f) Prove that $\delta_k^i \delta_u^k \delta_i^u = n$.

g) Show that, a symmetric tensor of order 2 has at most $\frac{n(n+1)}{2}$ different components.

h) Prove that $[i\,j, k] = [ji, k]$.

3. Answer any **two** questions: 5×2=10

a) Find the equation of integral surface given by differential equation

$$2y(z-3)p + (2x-z)q = y(2x-3),$$

which passes through the circle, z=0, $x^2+y^2=2x$. 5

b) Use convolution theorem to show that,

$$L^{-1}\left\{\frac{1}{(p+2)^2(p-2)}\right\} = \frac{1}{16}\left(e^{2t} - 4te^{-2t} - e^{-2t}\right).$$

5

c) If $\left(\frac{x^1}{x^2}, \frac{x^2}{x^1}\right)$ is a covariant vector in Cartesian co-ordinate system (x^1, x^2) then find its components in polar co-ordinates (r, θ). 5

4. Answer any **one** question: 10×1=10

a) i) Solve the P.D.E. xp+yq=pq by Charpits method.

ii) Solve the P.D.E. $p\tan x + q\tan y = \tan z$.

5+5=10

b) i) Solve by Laplace method the I.V.P.

$y''(t) + 25y(t) = 10\cos 5t$, given that $y(0)=2$, $y'(0) = 0$.

ii) Using convolution theorem for inverse Laplace transform, deduce that

$$\int_0^1 t^{a-1}(1-t)^{b-1}\,dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)};\ a>0,\ b>0.$$

5+5=10

c) i) Prove that Kronecker delta δ^i_j is a mixed tensor of rank 2.

ii) Show that in the Riemannian space E^4 with line element

$$(ds)^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2,$$

the vector $\left(1,\ 1,\ 0,\ \frac{\sqrt{3}}{c}\right)$ is a unit vector.

5+5=10
