U.G. 4th Semester Examination - 2021

PHYSICS

Course Code : BPHSCCHC 401

Course Title : Mathematical Physics III

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

a) Calculate i^{49} and i^{123} where $i^2 = -1$.

b) Find modulus of $\frac{1+2i}{1-(1-i)^2}$ where $i^2 = -1$.

c) Find the smallest value of positive integer, n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ where $i^2 = -1$.

d) Show that the product of two symmetric matrices of same order is symmetric, if the product be commutative.

e) Find the eigen values of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

f) Show that the matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ is unitary.

g) What is adjoint of a square matrix?

h) Examine whether $\overline{z}$ is an analytic function of z or not.

i) Write some examples of complex matrices which we often use in Physics.

j) What is complex conjugate of a matrix? Give example.

k) What is integral transformation? Give examples.

l) Show that Fourier transform is linear.

m) Write down the convolution property of Fourier transform.

n) If F(s) is the Fourier transform of f(x), then show that $F\{f(ax)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$.

o) Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$.

2. Answer any **five** questions: 2×5=10

a) Show that all the diagonal elements of a skew-Hermitian matrix are either zero or purely imaginary.

b) Show that $\mathrm{Tr}[AB]=\mathrm{Tr}[BA]$.

c) Expand the following function in a Laurent series about the point $z = 0$.

$$f(z)=\frac{1-\cos z}{z^3}$$

d) Evaluate the integral $\int_C \frac{e^z(z^2+1)}{(z-1)^2}dz$, where c is the circle $|z|=2$.

e) Find the residue of $e^{-\frac{1}{z}}$ at its singularity.

f) If f(z) is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2.$$

g) If F(s) be the Fourier transform of f(x), then show that

$$F\{f(x)\cos ax\}=\frac{1}{2}\{F(s+a)+F(s-a)\}.$$

h) Show that the Fourier transform of $\frac{df(x)}{dx}$ is $-ik\,F(k)$, where F(k) is the Fourier transform of f(x).

3. Answer any **two** questions: 5×2=10

a) Solve $x^4+i=0$, where $i^2=-1$. Write down necessary condition for a complex function $f(z)$ to be analytic. 3+2=5

b) Using Fourier transform, solve one-dimensional wave equation

$$\frac{\partial^2 y(x,t)}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2},\ \text{given } y(x,0)=f(x).$$

c) Determine eigenvalues and normalized eigenvectors of the following matrix

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & -2 & 0 \\ 2 & 7 & 3 \end{bmatrix}.$$

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