U.G. 6th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMGERT10A

Course Title : Basics of Higher Mathematics-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Find the identity element in $(\mathbb{Z}, +)$.

b) When a differential equation is said to be exact?

c) Find the general solution of the differential equation $y'' - 3y' + 2y = 0$.

d) Find the gradient of the lines joining the points on the curve $y = 3x^2 - 2x + 1$ whose abscissae are –1 and 2.

e) Define subgroup of a group $(G, \circ)$.

f) Examine whether the DE $y^2dx + 2xydy = 0$ is exact or not.

g) Show that the points (1, 1), (5, –9) and (–1, 6) are collinear.

h) Find the equation of the right bisector of the line joining the points (2, 3) and (4, 5).

i) If R be a non-trivial ring with unity I then $0 \neq I$.

j) What is the condition of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ to be perpendicular?

k) Define divisors of zero in a ring R.

l) Find the equation of the plane passing through $(2, -3, 4)$ and parallel to the plane $2x - 5y - 7z + 15 = 0$.

m) When a group $(G, \circ)$ is said to be abelian?

n) Write down the form of Clairaut's equation.

o) Find the equation of the line passing through the point (3, 2, –6) and perpendicular to the plane $3x - y - 2z + 2 = 0$.

2. Answer any **five** questions: 2×5=10

a) Solve the equation

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0.$$

b) In a group $(G, \circ)$, $(a \circ b)^2 = a^2 \circ b^2$ holds $\forall a, b \in G$. Prove that the group is abelian.

c) Find the equation of the line passing through the point (1, 2, 3) and perpendicular to the planes $x - 2y - z + 5 = 0$ and $x + y + 3z + 6 = 0$.

d) Find the slope of the straight line $\frac{l}{r} = \cos(\theta - \alpha) + e\cos\theta$.

e) Show that the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

f) Solve the differential equation $p = \log(px - y)$, where $p = \frac{dy}{dx}$.

g) Find the shortest distance from the point $(2, -7)$ to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$.

h) Prove that in a ring R with unity 1, $-1.a = -a, \ \forall a \in \mathbb{R}$.

3. Answer any **two** questions: $5 \times 2 = 10$

a) Find the reduction formula for the curve $f(x) = \sin^n x$.

b) Find λ so that the equation $x^2 + 5xy + 4y^2 + 3x + 2y + \lambda = 0$ represents a pair of straight lines. Find also the point of intersection and the angle between them.

c) Find the general solution and particular integral of the differential equation $y'' + 2y' + 2y = 10\sin 4x$.

4. Answer any **one** question: $10 \times 1 = 10$

a) i) Reduce the differential equation $(px - y)(px + y) = 2p$, where $p = \frac{dy}{dx}$, to Clairaut's form and solve.

ii) If the lines $ax^2 + 2hxy + by^2 = 0$ be the two sides of a parallelogram and the line $lx + xy = 1$ be one of the diagonals, show that the equation of the other diagonal is $y(bl - hm) = x(am - lh)$. Show that the parallelogram is a rhombus if $h(a^2 - b^2) = (a - h)lm$. 5+5

b) i) Obtain the equations to the sphere through the common circle of the sphere $x^2 + y^2 + z^2 + 2x + 2y = 0$ and the plane $x + y + z + 4 = 0$ which intersects the plane $x+y=0$ in circle of radius 3 units.

ii) Let $M_2(\mathbb{R})$ be the set of all 2×2 matrices whose elements are real numbers. Prove that the set $M_2(\mathbb{R})$ forms a commutative group with respect to 'matrix addition (+)'.

5+5

c) i) If 2d be the shortest distance between the lines $x=0, \frac{y}{b}+\frac{z}{c}=1$ and $y=0, \frac{x}{a}-\frac{z}{c}=1$ then prove that, $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$.

ii) Solve the differential equation

$$(2x+\tan y)dx+(x-x^2\tan y)dy=0$$

by finding an integrating factor. 5+5

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