2021

PHYSICS

[HONOURS]

Paper : VI

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any **ten** questions: 2×10=20

1. a) In a Compton Scattering experiment, photons with incoming momentum mc (m is mass of the electron) are scattered at angle 90^o. What is the magnitude of the momentum of the scattered photon?

   b) Write down physical significance of a wave function Ψ.

   c) Find $\left[x, p_x^2\right]$.

   d) For a Gaussian wave packet

   $\psi(x) = A \exp\left(-x^2/a^2\right)$, find the momentum operator.

e) What is probability current density? Write down the expression of probability current density in 3D.

f) Show that Hermitian operator is always linear.

g) Calculate the de Broglie wavelength of an electron of energy 1GeV.

h) Prove that : $\left(\frac{d}{dx}\right)^+ = -\frac{d}{dx}$.

i) Find the fraction of volume unoccupied in the unit cell of the b.c.c. latties.

j) Find the Miller indices of the plane parallel to x and y axes.

k) The first order (100) reflection angle is 30^o for a cubic crystal using X-rays of wavelength 1.54Å. Determine the distance between the (111) planes of the crystal.

l) Explain acoustic and optic modes of vibration.

m) The Debye temperature of a metal is 450K. Calculate its Debye frequency in Hz.

n) Define the Curie law for paramagnetism. What is Curie temperature?

o) The static dielectric constant of water is 8.1 and its R.I. is 1.33. Calculate % contribution of ionic polarisability.

GROUP–A

(Quantum Mechanics I)

Answer any **two** questions: 5×2=10

2. Consider an X-ray beam with wavelength λ scattered with free electron. If $\Delta\lambda$ is change in wavelength of X-ray after scattering from electron of mass m then prove that change in K.E. of recoiling electron is $\frac{hc}{\lambda}\frac{\Delta\lambda}{(\lambda+\Delta\lambda)}$.

3. Use the uncertainty principle to estimate the radius and ground state energy of the hydrogen atom.

4. Show that the probability current density $\hat{j}$ is defind by

$$\hat{J} = i\frac{h}{2m}\left(\Psi\nabla\psi^* - \psi^*\nabla\psi\right)$$

as a consequence of conservation of probability. Can $\hat{j}$ be imaginary?

5. Calculate the probability of transmission of α-particle through a rectangular barrier V_o=2eV, E=1eV, berrier width = 1Å.

Answer any **one** question: 10×1=10

6. Calculate the reflection and transmission coefficient for a stream of particles each of mass 'm' and energy E incident on a rectangular potential. ($E < V_o$)

5+5

7. A particle of mass m, which moves freely inside an infinite potential well of length a, is initially in the state $\Psi(x,0) = \sqrt{\frac{3}{5a}}\sin\frac{3\pi x}{a} + \frac{1}{\sqrt{5a}}\sin\frac{5\pi x}{a}$

a) Find $\Psi(x,t)$ at any later time t.

b) Calculate the probability density $\rho(x, t)$ and the probability current density J(x,t).

c) Verify that the probability is conserved i.e $\frac{\partial\rho}{\partial t} + \vec{\nabla}.\vec{J}(x,t) = 0$. 2+(3+3)+2

GROUP–B

(Quantum Mechanics II)

Answer any **two** questions: 5×2=10

8. A particle of mass m moving along x axis which is acted upon by a constant potential V_0 for $x > 0$. Write down the Schrodinger equation for the particle. What happens if $V_0 > E$, where E is the energy of the particle.

9. A particle moves in a potential field
$V(x, y) = \frac{1}{2}mw^2(x^2+y^2) - bx$. where b = constant.
Find the ground state energy.

10. For an one-dimensional S.H.O show that
$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$
$\hat{a}^+|n\rangle = \sqrt{n+1}|n+1\rangle$
symbols are conventional.

11. Prove that: $\hat{L}_{+}=\hat{L}_{x}+i\hat{L}y=he^{i\phi}\left(\frac{\partial}{\partial\theta}+icot\theta\frac{\partial}{\partial\phi}\right)$.

Answer any **one** question: $10\times1=10$

12. a) If the operators A and B satisfy $\left[\hat{A},\hat{B}\right]=\hat{I}$, prove that $\left[e^{\hat{A}},\hat{B}\right]=e^{\hat{A}}$.

    b) If $\left[\hat{x},\hat{p}\right]=ih$ find $\left[\hat{x},\hat{p}e^{-\hat{p}}\right]$ in natural units $(h=1)$. 5+5

13. Write down the time-independent Schrödinger equation of Hydrogen atom (in spherical polar coordinate). Find the θ equation, Φ-equation and radial equation by separation of variable. Solve the radial equation and find $\Psi_{nl}(r)$. Plot a graph of probability distribution for n = 1 and l = 0. 2+2+4+2

GROUP–C

(Solid State Physics I)

Answer any **two** questions: $5\times2=10$

14. Prove that any reciprocal lattice vector $G_{hkl}=h\vec{a}^{*}+k\vec{b}^{*}+l\vec{c}^{*}$ is always perpendicular to the set of direct lattice plane (hk$l$). The magnitude of this vector $\left|G_{hkl}\right|=\frac{1}{d_{hkl}}$ where d_{hkl} denotes the inter planar spacing of that (hk$l$) plane. 3+2

15. If the potential energy $\phi(r)=\frac{-\alpha}{r^{6}}+\frac{\beta}{r^{12}}$.

    Find the internuclear distance for which potential energy is zero. Also show that minimum potential energy is $\phi_{0}=-\frac{\alpha^{2}}{4\beta}$. 5

16. How does a classical free electron theory lead to the Ohm's law? Find out the expression for Hall co-efficient for a metal. 2+3

17. Define direct and indirect band gap semiconductor with proper band diagram. Taking the origin at the bottom of the conduction band. Calculate the crystal momentum for a free electron of energy 0.02 eV. Given effective mass of the electron = 0.2 m_{0}. 2+3

Answer any **one** question: $10\times1=10$

18. Define different types of point group symmetry operations. Specify all the point group symmetry operations for a simple cubic lattice structure. Prove that five fold rotational symmetry cannot exist in nature. 3+3+4

19. Deduce Lau equations and explain Ewald construction.

GROUP–D

(Solid State Physics II)

Answer any **two** questions: 5×2=10

20. Derive an expression for orientational polarizability using Langevin's theory. 5

21. Prove theat $X_{dia} = -NZ\left(\frac{e^2}{6mc^2}\right)r^2$ where the symbols have usual meanings. 5

22. Explain the difference between Type-I and Type-II superconductors using the Meissner effect. Describe the isotope effect in superconductors. 3+2

23. What is high T_c superconductivity? Mention two compounds exhibiting the above effect.

    The critical temperature T_c for Hg with isotopic mass 199.5 amu is 4.18K. Calculate its critical temperature when its isotopic mass changes to 203.4 amu. 1+1+3

Answer any **one** question: 10×1=10

24. Derive the vibrational modes of diatomic linear chain of atoms. What is the difference between the two branches? Why are they so named? 6+2+2

25. Deduce Debye T_3 law for a solid. What is the significance of Debye temperature? Write down the limitations of Debye's theory. 5+2+3

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