U.G. 6th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMDSRT-3 & 4 (DSE 3 & 4)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

This question papers contains both DSE 3 & 4. Students are thereby instructed to answer DSE paper out of these two (DSE 3 & DSE 4) as he/she opted for.

Title : Probability and Statistics

Code : BMTMDSRT3 (DSE 3)

1. Answer any **ten** questions: 1×10=10

   a) Write down the classical definition of probability.

   b) Set up a sample space for the three times toss of a coin.

   c) Define probability distribution function for a random variable X.

   d) **True** or **False**: Variance is the second central moment.

   e) What is the minimum value of standard deviation?

   f) Consider the function given by

$$f(x)=\begin{cases} kx(1-x), & \text{if } 0\le x\le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   If f(x) defines a probability density function, find the value of k.

   g) What do you mean by a random sample?

   h) Write down the probability mass function of binomial (n, p) variate.

   i) Find the first moment about the point 5 for the set of numbers 4, 6, 8, 10.

   j) Let X be a continuous random variable with probability density function f(x). If C is any constant find P(X=C).

   k) Choose the correct answer:

   The correlation co-efficient lies between

   i) 1 and 2

   ii) 0 and 1

iii) -1 and 1

iv) $-\frac{1}{2}$ and $\frac{1}{2}$

l) Define marginal density functions for a two dimensional continuous random variable (X, Y).

m) What is a scatter diagram?

n) State two properties of maximum likelihood estimators.

o) State Tchebycheff's Inequality.

2. Answer any **five** questions: $2\times5=10$

a) A discrete random variable X has the following probability mass function:

$x_i = i$:	-3	-2	-1	0	1	2
$P(X=i) = f_i$:	k	$2k$	$2k^2$	$3k^2$	k^2	$6k^2+8k$

Find the value of k.

b) For any random variable X, show that $\text{Var}(aX+b)=a^2\text{Var}(X)$, where a and b are real constants.

c) Out of the two lines given by 3x+9y=46 and 3y+12x=19 which one is the regression line of x on y?

d) Let X be binomial (n, p) variate. Find the moment generating function $M_X(t)$ of X.

e) Let X be a discrete random variable whose spectrum is the set $\left\{\frac{3^k}{k^2} : k=1, 2, 3, \ldots\right\}$ and the probability mass function of X be given by $P\left(X=\frac{3^k}{k^2}\right)=\frac{2}{3^k}$, k=1, 2, 3, ... Does E(X) exist?

f) If X, Y are two independent random variables, prove that X, Y are uncorrelated, i,e, $\rho(X, Y)=0$, provided $\rho(X, Y)$ exists.

g) The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. At the time of checking it was found that one observation 8 is incorrect. Calculate the mean and standard deviation if the wrong observation is ommited.

h) What do you mean by a null hypothesis and an alternative hypothesis?

3. Answer any **two** questions: $5\times2=10$

a) i) A continuous random variable X follows the probability law $f(x)=\begin{cases}\frac{3}{10}x^2, & 0\le x\le 1\\ 0, & \text{elsewhere.}\end{cases}$

Compute $P\left(\frac{1}{4}<X<\frac{1}{2}\right)$.

ii) Let $X_1, X_2, ..., X_n$ be a random sample from a population. Define sample mean and sample variance. $3+2=5$

b) Let X be the random variable having normal (m, σ) distribution. Write down the probability density function f(x) of X. Show that any odd order central moment of X is zero. $1+4=5$

c) i) Describe Bernoulli Trials with an example.

ii) A random variable X has the following discrete distribution:

x_i:	-3	-2	-1	0	1	2	3	4
f_i:	$\frac{1}{50}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{7}{10}$	$\frac{1}{100}$

Find the distribution of Y, where $Y=X^2$. $2+3=5$

4. Answer any **one** question: $10\times1=10$

a) i) A two dimensional random variable (X, Y) has the spectrum $(x_i, y_j)=(i, j);\ (i=0, 1, 2, 3;\ j=1, 2, 3, 4)$ and the joint probabilities f_{ij} are given by $f_{ij}=P(X=i, Y=j)=c(3i+4j)$, where c is a constant. Find the value of c. 4

ii) The following values relate to bivariate data on (x, y):

$\sum_i x_i=120$, $\sum_i y_i=90$, $\sum_i x_i^2=600$, $\sum_i y_i^2=300$, $\sum_i x_i y_i=414$, n=30.

Find the correlation co-efficient between x and y. 4

iii) If a two dimensional continuous random variable (X, Y) is normally distributed, write down its joint density function f(x, y). 2

b) i) Two random variables X and Y have the following joint probability density function:

$f(x, y)=\begin{cases}2-x-y; & 0\le x\le 1,\ 0\le y\le 1\\ 0, & \text{elsewhere.}\end{cases}$

Find marginal density functions of X and Y. Hence find var(X) and var(Y). 5

ii) Suppose we draw two observations X_1 and X_2 at random from normal $N(\mu, \sigma^2)$. If we define an estimator of μ as $T = \frac{1}{3}X_1 + \frac{2}{3}X_2$, prove that T is an unbiased estimator of μ. 5

c) i) Define two dimensional expectation. Let (X, Y) be a two dimensional random variable with joint probability density function given by

$$f(x, y) = \begin{cases} xe^{-x(y+1)}; & x \geq 0,\ y \geq 0 \\ 0, & \text{elsewhere}. \end{cases}$$

Compute E(Y|X=x). 5

ii) Let the continuous random variable X be uniformly distributed in the interval (a, b), where $-\infty < a < b < \infty$. The probability density function of X is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a < x < b \\ 0, & \text{elsewhere}. \end{cases}$$

Show that $E(X) = \frac{b+a}{2}$ and $Var(X) = \frac{(b-a)^2}{12}$. 5

Title : Mechanics-II

Code : BMTMDSRT4 (DSE 4)

1. Answer any **ten** questions: $1\times10=10$

a) How does a rigid body differ from a deformable body?

b) When a fluid called non-homogeneous?

c) What is the resultant of a force and couple?

d) Define a catenary of uniform strength.

e) Define equi-density surface of a fluid.

f) State the principle of virtual works for a particle.

g) What do you mean by incompressible fluid?

h) Write down the conditions of equilibrium of a system of non-coplanar forces.

i) State Archimedes' principle for a floating body.

j) Define the centre of pressure of a plane area immersed in a heavy homogeneous liquid.

k) Write down stress matrix of perfect fluid.

l) Define a stress vector at a point of a continuous medium.

m) Can a force and a couple in the same plane produce equilibrium?

n) Write down the equation of common catenary.

o) Can a fluid be simultaneously viscus and compressible?

2. Answer any **five** questions: $2\times5=10$

a) What is Poinsot's central axis? Write down its equation.

b) State the energy test of stability.

c) State the converse of the principle of virtual work.

d) Write down the pressure-density relation for a perfect gas in

i) isothermal change of state

ii) adiabatic change of state

e) State the necessary and sufficient conditions for equilibrium of a rigid body under the action of a system of external forces.

f) Define stress vector in a continuum. What is a stress matrix at a point in continuum?

g) When is the equilibrium of a heavy body resting on a fixed rough body said to be

i) stable

ii) neutral?

h) Define deformable body. When is it called

i) elastic

ii) fluid?

3. Answer any **two** questions: $5\times2=10$

a) A square lamina rests with its plane perpendicular to a smooth wall. One corner being attached to a point in the wall by a finite string of length equal to the side of the square. Find the position of equilibrium and show that it is stable.

b) A string of length $2l$ feet is suspended between two points at the same level and the lowest point of the string is d feet below the points of suspension. If the weight per unit length of the string be w lbs, find the horizontal component of the tension.

c) Forces X, Y, Z act along the three straight lines $y=b,\ z=-c;\ z=c,\ x=-a$ and $x=a,\ y=-b$ respectively. Show that they will reduce to a single resultant force, if $\frac{a}{X}+\frac{b}{Y}+\frac{c}{Z}=0$. Find the equation of the central axis.

4. Answer any **one** question: 10×1=10

a) i) Two uniform rods, each of weight 'w' and length'a' are freely jointed at 'A' and each passes over a smooth peg at the same level. From 'A' weight W' is suspended. Show that in the position of equilibrium the inclination θ of each rod to the horizon is given by $\cos^3\theta = \frac{c(2W+W')}{2Wa}$, c being the distance between the pegs.

ii) Define central axis and derive its equation.

b) i) A force P acts along the axis of x and another force nP acts along a generator of the cylinder $x^2+y^2=a^2$. Show that the central axis lies on the cylinder $n^2(a-z)^2+(1+n^2)^2 y^2 = n^4a^2$.

ii) If the components parallel to the axes of the forces acting on an element of the fluid at (x, y, z) be proportional to $y^2+2lyz+z^2$, $z^2+2\mu zx+x^2$, $x^2+2vxy+y^2$, show that, if equilibrium be possible, then $2l=2\mu=2v=1$.

c) i) Find the virtual work done by the mutual action and reaction between two particles.

ii) A vessel having a plane base and plane vertical sides, contains two liquids which do not mix. Find the resultant thrust on one of the sides.

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