U.G. 1st Semester Examination - 2020

BCA

Course Code : BBCAGEHT103

Course Title : Mathematics-I

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

a) Find the equation whose roots are 1, –2, 3, –4.

b) If A={1,2,3,4,5} B={4,5,6,7,8} then find the value of $A\Delta B$, where A and B are sets.

c) Find the modulus and amplitude of (5+12i).

d) Find the relation between a and b, if $\left(ax^5+3bx^3+8\right)$ be exactly divisible by (x–2)

e) Find the transformed equation of the st. line $x\cos\alpha+y\sin\alpha=\rho$ when the axes are turned through an angle α without any change of origin.

f) Find the nature of the conic $\frac{8}{r}=4-5\cos\theta$.

g) Express (–1,2,4) as a linear combination of $\bar{\alpha}=(-1,2,0)\,\&\,\bar{\beta}=(0,-1,1)$.

h) Find the value of $\begin{vmatrix} 0 & -2 & 1 \\ 3 & 4 & 0 \\ 1 & 2 & 4 \end{vmatrix}$.

i) Define orthogonal matrix. Is the matrix $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ is orthogonal?

j) If R is a relation from a finite set A having 3 elements to a finite set B having 5 elements then, find the number of relations from A to B.

k) Find the general value of i^i.

l) The set S={1, –1, –i, i} from an Abelian group w.r. to multiplication. Then find O(i), O(–1).

m) If $x+\frac{1}{x}=2\cos\frac{\Pi}{7}$, then show that $x^7+\frac{1}{x^7}=-2$.

n) If ω be a cube root of unity then find the value of $\omega^{2018}+\omega^{2019}+\omega^{2020}$.

o) What does the equation $x^2+y^2=9$ represents geometrically?

2. Answer any **five** questions: $2\times5=10$

a) Find the sum of 99th powers of the roots of the equation $x^7-1=0$.

b) For what value of λ, the equation $x^2+\lambda xy-2y^2+3y-1=0$ represent a pair of straight lines?

c) If $|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5$ such that each of them are perpendicular to sum of the others two, then find $|\vec{a}+\vec{b}+\vec{c}|$.

d) If n be a positive integer, then prove that $(1+i)^n+(1-i)^n=2^{\frac{n}{2}+1}\cos\frac{1}{4}n\pi$.

e) If α,β,γ be the roots of the cubic equation $x^3+px^2+qx+r=0$, find the value of $\Sigma\frac{1}{\alpha\beta}$.

f) Find the condition that the cubic $x^3-px^2+qx-r=0$ should have its roots in G.P.

g) Find the equation of the directrix of the conic $r\sin^2\frac{\theta}{2}=1$.

h) Determine a unit vector perpendicular to the plane of vectors $\overline{A}=2\hat{i}-6\hat{j}-3\hat{k}$ and $B=4\hat{i}+3\hat{j}-\hat{k}$.

3. Answer any **two** questions: $5\times2=10$

a) Let A, B, C be subsets of S then prove that $n(A\cup B)=n(A)+n(B)-n(A\cap B)$ [using Venn diagram].

b) If α,β,γ be the roots of the equation $x^3+2x^2+3x+4=0$, then find the equation whose roots are $1+\frac{1}{\alpha}, 1+\frac{1}{\beta}, 1+\frac{1}{\gamma}$, also find the value of $\left(1+\frac{1}{\alpha}\right), \left(1+\frac{1}{\beta}\right), \left(1+\frac{1}{\gamma}\right)$.

c) Solve by Cardan's method $x^3-9x+28=0$.

4. Answer any **one** question: $10\times1=10$

a) i) If the pairs of lines represented by $x^2-2cxy-y^2=0$ & $x^2-2dxy-y^2=0$ be such that each pair bisects the angle between the other pair, prove that cd=–1.

ii) Find the value of the constant a, such that the vectors $\left(2\hat{i}-\hat{j}+\hat{k}\right),\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ & $\left(3\hat{i}+a\hat{j}+5\hat{k}\right)$ are coplanar.

iii) Reduce the equation $3x^2+2xy+3y^2-16x+20=0$ to its standard form and hence show that it is an ellipse. 3+3+4=10

b) i) Solve by Cramer's

$$x+2y-3z=1$$
$$2x-y+z=4$$
$$x+3y=5$$

ii) Find the principal and general value of $(1+i)^i$. 7+3=10

c) i) Show that the vectors (1,2,1), (2,1,0), (1,–1,2) form a basis of R^3.

ii) Show that the set Z, of all integers form a group under the binary operation * defined by a*b = a+b+1, $a, b \in z$.

5+5=10

―――――――――