U.G. 1st Semester Examination - 2020

MATHEMATICS

Course Code : BMTMCCRT101

Course Title : Calculus & Analytical Geometry (2D)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions from the following: $1\times10=10$

a) If $y=x^{-x}$, then find $\frac{d^2y}{dx^2}$ at $x=1$.

b) Write down the n-th derivative of $y=e^{ax}$ w.r.to x.

c) Evaluate: $\lim_{x\to1}\frac{\log x}{x-1}$.

d) If $z=\log\left(x^2+y^2\right)$, then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

e) State Euler's theorem for a homogeneous function f(x,y).

f) Find the Jacobian $J=\frac{\partial(x, y)}{\partial(r, \theta)}$, where $x=r\cos\theta$, $y=r\sin\theta$.

g) Find the area of the region bounded by the curves $y=x^2$ and $x=y^2$.

h) Evaluate the value of $\int_0^{\frac{\pi}{2}}\sin^5\theta\, d\theta$.

i) Show that the equation $2x^2+3xy+y^2=0$ represents a pair of straight lines.

j) Find the centre and radius of the circle $8x^2+8y^2-12x+20y-1=0$.

k) Determine the nature and length of latus rectum of the conic $r(3-3\cos\theta)=2$.

l) Transfer the polar equation $\theta=\alpha$ (α is constant) into cartesian equation.

m) Find the equation of tangents to the curve $x^3+y^3=3axy$ at origin.

n) Show that the curve $y=\log x$ is convex in $0<x<1$ with respect to x-axis.

o) If $y=Ae^{2x}+Be^{-2x}$ (A, B are constants), prove that $\frac{d^2y}{dx^2}=4y$.

2. Answer any **five** questions: 2×5=10

a) If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, then find $\frac{d^2y}{dx^2}$.

b) Find the radius of curvature at any point (x, y) to the curve $xy=c^2$ (c is a constant).

c) Obtain the pedal equation to the polar curves $r\theta = a$ with respect to pole.

d) Find the angle through which the axes are to be rotated so that the equation $x^2 + 2\sqrt{3}xy - y^2 = 2$ may be reduced to the form $(x')^2 - (y')^2 = 1$.

e) Prove that the bisectors of the angles between the pair of straight lines $ax^2+2hxy+by^2=0$ and $a'x^2 + 2h'xy + b'y^2 = 0$ are the same if $h(a' - b') = h'(a - b)$.

f) Prove that the length of the focal chord of the conic $\frac{l}{r} = 1 - e\cos\theta$ which is inclined to the axis at angle α is $\frac{2l}{1 - e^2\cos^2\alpha}$.

g) Obtain reduction formula for $\int \tan^n x\, dx$, n is a positive integer greater than 1.

h) Find the length of the circumference of the circle $x^2+y^2=36$.

3. Answer any **two** questions: 5×2=10

a) If $y = \sin(m\sin^{-1}x)$, show that

i) $(1 - x^2)y_2 - xy_1 + m^2y = 0$

ii) $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$.

b) If $J_n = \int_0^{\frac{\pi}{2}} \sin^n x\, dx$, then show that $J_n = \frac{n-1}{n} J_{n-2}$, where $n > 1$ is a positive integer.

c) Prove that the straight line $lx+my+n=0$ touches the parabola $y^2 = 4p(x - q)$ if $l^2q + ln = pm^2$.

4. Answer any **one** question: 10×1=10

a) i) Reduce the equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$, to its canonical form.

ii) Show that y=x is one of the bisectors of the angle between the pair of straight lines $\sqrt{3}(x^2 + y^2) - 4xy = 0$. 7+3

b) i) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$.

ii) Show that the sum of the intercepts of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ on the axes is a constant.

iii) Find the vertical asymptotes to the curve $y = \tan x$. 4+4+2

c) i) Find the volume of the solid generated by revolving the cardioide $r = a(1 - \cos\theta)$ about the initial line.

ii) Show that the envelope of the ellipses $\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$, where the parameters α, β are connected by, $\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$ is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$.

iii) Show that $y=x^4$ is concave upwards at (0, 0). 4+4+2

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