U.G. 1st Semester Examination - 2020
PHYSICS

Course Code : BPHSCCHC 101

Course Title : Mathematical Physics

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: $1\times10=10$

a) Find the area of a parallelogram whose adjacent sides are $\hat{i}-2\hat{j}+3\hat{k}$ and $2\hat{i}-\hat{j}+4\hat{k}$.

b) Prove that: $\left(\vec{A}\times\vec{B}\right)^2=A^2B^2-\left(\vec{A}\cdot\vec{B}\right)^2$.

c) What is a homogeneous differential equation of first order?

d) Suppose $\vec{A}(2)=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{A}(3)=4\hat{i}-2\hat{j}+3\hat{k}$

Evaluate: $\int_2^3 \vec{A}\cdot\left(\frac{d\vec{A}}{dt}\right)dt$.

e) Show that the differential $xdy + 3ydx$ is inexact.

f) Plot $3\sin^2 x$ vs x.

g) Find the unit vectors of a cylindrical co-ordinate system in terms of $\hat{i}$ and $\hat{j}$.

h) Two particles have velocities $\vec{v}_1=\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{v}_2=\hat{i}-2\hat{k}$, respectively. Find the velocity $\vec{u}$ of the second particle relative to the first.

i) Find the complementary function of the equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x$.

j) Express ∇ operator in generalized curvilinear coordinates.

k) Solve $(2x-y+1)dx + (2y-x-1)dy = 0$

l) Give some practical application of time dilation.

m) What do you mean by vector field?

n) What is the difference between directional derivative and normal derivative?

o) What is Lagrange multiplier used for?

2. Answer any **five** questions: $2\times5=10$

a) Prove that:

$$\hat{i}\times(\hat{a}\times\hat{i})+\hat{j}\times(\hat{a}\times\hat{j})+\hat{k}\times(\hat{a}\times\hat{k})=2\hat{a}.$$

b) Prove that: $(y^2-z^2+3yz-2x)\hat{i} + (3xz+2xy)\hat{j} + (3xy-2xz+2z)\hat{k}$ is both solenoidal and irrotational.

c) A vector field $\hat{F}$ is given by $\vec{F}=\sin y\,\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral $\int_C \vec{F}.d\vec{r}$ where C is the circular path given by $x^2+y^2=a^2$.

d) Evaluate $\int \vec{r}\cdot\hat{n}\,ds$ over the unit cube defined by the point (0, 0, 0) and the unit intercepts on positive x, y, z axis.

e) Show that scalar product is invariant under rotation.

f) Expand e^x in powers of $(x-1)$ upto four terms.

g) Given that $x(u) = 1 + au$ and $y(u) = bu^3$, find the rate of change of $f(xy) = xe^{-y}$ with respect to you.

h) Solve $(x+1)\dfrac{dy}{dx} - y = e^{3x}(x+1)^2$.

3. Answer any **two** questions: $5\times2=10$

a) Find the divergence and curl of $\vec{v} = xyz\,\hat{i} + 3x^2y\,\hat{j} + (xz^2-y^2z)\hat{k}$ at (2, –1, 1). If a force $\vec{F} = 2x^2y\,\hat{i} + 3xy\,\hat{j}$ displaces a particle in the xy plane from (0, 0) to (1, 4) along a curve $y = 4x^2$. Find the work done. 3+2=5

b) Apply Gauss divergence theorem to evaluate the following integral, $\iint_S \vec{F}.\vec{ds}$, where S is the surface of the sphere $x^2+y^2+z^2=16$ and $\vec{F} = 3x\,\hat{i}. + 4y\,\hat{j} + 5z\,\hat{k}$. Write down Laplacian operator in spherical polar coordinate.

3+2=5

c) Solve the following differential equation using integrating factor technique

$$x(x^2-1)\frac{dy}{dx}-(3x^2-1)y=x^5-2x^3+x.$$

The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1+xy$. Find the temperature of the two hottest points on the circle.

$2\frac{1}{2}+2\frac{1}{2}=5$