U.G. 5th Semester Examination - 2020

PHYSICS

Course Code : BPHSDSHC1 [DSE 1]

Course Title : Advanced Mathematical Physics

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

a) Write down the condition that the vectors $\{|a_1\rangle, |a_2\rangle, ..., |a_n\rangle\}$ are to be linearly independent.

b) V is the vector space of all 2×2 real matrices. Prove that the set

$$S=\left\{\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}, \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right\}$$

is a basis of V.

c) Prove that the inner product space is positive definite.

d) Show that set of vectors

$$S=\{(3, 0, 4), (-4, 0, 3), (0, 9, 0)\}$$

is orthogonal.

e) What do you mean by vector space homomorphism?

f) Test the transformation $L:\mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$T(x, y)=(x, x+y, x-y)$$

is linear or not.

g) Show that the operator D, $Df(x)=\frac{d}{dx}f(x)$, is linear.

h) If A and B are linear operator then prove that $(AB)^{-1}=B^{-1}A^{-1}$.

i) Prove that the inner product of the vectors preserve under unitary transformation.

j) Give an example of a tensor of rank 2.

k) Write down the Euler's differential equation with N-independent variables.

l) In spherical polar coordinate system r, θ, φ are three independent coordinates (as usual).

The infinitesimal line eliment ds is given by $ds^2 = \eta_{ij} dx^1 dx^2$. What is η_{ij}?

m) Write down the metric tensor g_{ij} for Cartesian coordinate system.

n) Transform a covariant vector A_μ into a contravariant vector.

o) Solve the Euler's equation for $\int_{x_0}^{x_1} (x + y') y' dx$.

2. Answer any **five** questions: 2×5=10

a) Prove that in an n-dimensional vector space every set of (n+1) vectors is linearly dependent.

b) Show that the following three vectors in E^3 can not serve as base vectors of E^3, $|1\rangle = (1, 1, 2)$; $|2\rangle = (1, 0, 1)$; $|3\rangle = (2, 1, 3)$.

c) Show that the mapping $\mathrm{P} : \mathbb{R}^3 \to \mathbb{R}^3$, the projection of a vector into the xy plane, defined by $\mathrm{F}(\mathrm{x}, \mathrm{y}, \mathrm{z}) = (\mathrm{x}, \mathrm{y}, 0)$ is a singular transformation.

d) Prove that the eigenvectors corresponding to different eigen values of an Hermitian matrix are orthogonal.

e) If $g_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ find out g^{ij}. (Symbols have their usual meanings)

f) Consider a linear transformation $T : \mathbb{C}^2 \to \mathbb{C}^2$ such that $T(z_1, z_2) = (iz_1, (1+i) z_2 - z_1)$. Calculate matrix representation M_T with the basis $\{(i, 0), (0, 1)\}$ of $\mathbb{C}^2$.

g) A vector is defined as $X = x_i e_i$ with a basis $\{e_i\}$. Show that under change of basis $(e_i \to e_i')$, the component of the above vector in new basis (e_i') can be written as $x_i' = (S^{-1})_{ij} x_j$, where S_{ij} is the matrix of basis transformation. If the transformation is orthogonal then prove that $x_i' = (S^T)_{ij} x_j$, where superscript T denotes transpose.

h) Show that shortest distance over the surface of a sphere joining two points on the sphere is a great circle.

3. Answer any **two** questions: 5×2=10

a) Consider the two linearly independent vectors

$$|U\rangle = (3-4i)|1\rangle + (5-6i)|2\rangle$$

$$|W\rangle = (1-i)|1\rangle + (2-3i)|2\rangle,$$

where $|1\rangle$ and $|2\rangle$ are an orthonormal basis. Apply the Gram-Schmidt process to transform the two vectors into an orthonormal basis.

b) i) Consider the linear operator $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x, y) = (4x + 5y, 2x - y)$, find the matrix A representing the basis $S = \{u_1, u_2\} = \{(1, 4), (2, 9)\}$.

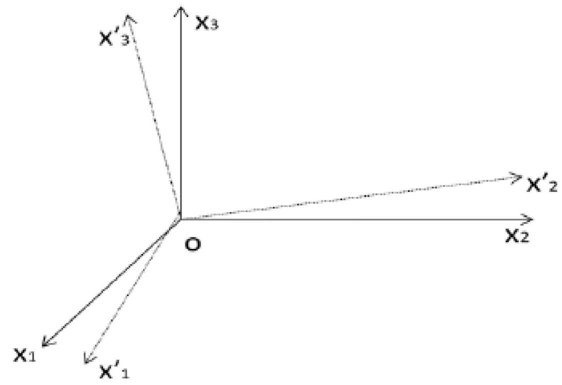
ii) Consider the following two bases of $\mathbb{R}^2$:

$S = \{u_1, u_2\} = \{(1, 2), (3, 5)\}$ and

$T = \{w_1, w_2\} = \{(1, -1), (1, -2)\}$,

find the change of basis matrix P from S to the new basis T. $2\frac{1}{2}+2\frac{1}{2}$

c) Suppose x, y, z is an orthogonal Cartesian co-ordinate system. x′, y′, z′ is a rotated coordinate system w.r.t. the origin O. The origin of both the co-ordinate systems are coincident. If $a_{ik} = \cos(x_i' x_k)$



Show that $x_i' x_j = a_{ik} a_{jl} x_k x_l$ then define the transformation property of a second rank tensor T_{ij}.

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