U.G. 3rd Semester Examination - 2020

MATHEMATICS

Course Code : BMTMCCRT301

Course Title : Geometry-3D and Vector Analysis

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Write the equation of the tangent plane to the sphere $x^2+y^2+z^2=14$ at the point (1,–2, 3).

b) Find the co-ordinates of the centre of the circle $x^2+y^2=4, z=10$.

c) Find the equation of the cone whose vertex is the origin and base is the circle $x=a, y^2+z^2=b^2$.

d) The plane z – 1 = 0 intersects the ellipsoid $\frac{x^2}{48}+\frac{y^2}{12}+\frac{z^2}{4}=1$ in an ellipse, determine the semi-axes.

e) What type of the surface does the following equation $x^2+3y^2-2z^2=0$ represent?

f) Determine the points of intersection of the line $\frac{x+2}{-1}=\frac{y+4}{-1}=\frac{z-3}{1}$ and the cylinder $x^2+z^2=1\cdot$

g) Find the equation of the sphere through the circle $x^2+y^2+z^2=9, 2x+3y+4z=5$ and the origin.

h) Show that the line $\frac{x}{2}=\frac{y}{1}=\frac{z}{-2}$ is a generator of the cone $xy-yz+zx=0$.

i) Find the equation of the enveloping cone of the ellipsold $4x^2+3y^2+7z^2=1$ with its vertex at (2, 1, 3).

j) Find $\vec{\nabla}\varphi$ where $\varphi=\log|\vec{r}|$.

k) Show that the vector $\vec{V}=yz\,\hat{i}+zx\,\hat{j}+xy\,\hat{k}$ is solenoidal.

l) Simplify $\frac{d}{dt}\left(\vec{r}\times\frac{d\vec{r}}{dt}\right)$.

m) If $\vec{r}=t^2\hat{i}-t\,\hat{j}+(2t+1)\hat{k}$; find the value of $\left|\frac{d\vec{r}}{dt}\right|$ at t = 0.

n) Find $\text{div}(\text{grad}\,\phi)$, where $\phi = 2x^2y^3z^4$.

o) Show that the vectors $(2\hat{i}-\hat{j}+\hat{k})$, $(\hat{i}+2\hat{j}-3\hat{k})$ and $(3\hat{i}-4\hat{j}+5\hat{k})$ are coplanar.

2. Answer any **five** questions: $2\times5=10$

a) Find the equation of the tangent planes to the hyperbolic paraboloid $5x^2-2y^2=2z$ parallel to $10x-6y-z=7$ and also find the point of contact.

b) Find the nature of the conicoid, $3x^2-2y^2-12x-12y-6z=0$.

c) Find the equation of a right circular cylinder whose axis is $\frac{x}{1}=\frac{y}{0}=\frac{z}{-2}$ and radius equal to 7 units.

d) Find the equation of sphere which passes through the origin and touches the sphere $x^2+y^2+z^2=56$ at the point (2,–4,6).

e) Show that the quadric surface given by the equation $2x^2+5y^2+3z^2-4x+20y-6z-5=0$ represents an ellipsoid with centre (1,–2,1).

f) If $\vec{r}=\cos nt\,\hat{i}+\sin nt\,\hat{j}$ where n is a constant and t varies; then show that $\vec{r}\times\frac{d\vec{r}}{dt}=n\hat{k}$.

g) Find the divergence of the vector $xy\sin z\,\hat{i}+y^2\sin x\,\hat{j}+z^2\sin xy\,\hat{k}$ at $\left(0,\frac{\pi}{2},\frac{\pi}{2}\right)$.

h) Calculate the normal derivative of $f(x,y,z)=xy+yz+zx$ at the point (–1, 1, 1).

3. Answer any **two** questions: $5\times2=10$

a) Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.

b) Reduce the equation $3x^2-y^2-z^2+6yz-6x+6y-2z-2=0$ to its canonical form.

c) If $\vec{F}(x,y,z)=(x^2+y^2)\hat{i}+(6-3xy)\hat{j}+4\hat{k}$; evaluate $\iint_S \vec{F}.\vec{n}\,ds$ where S denotes the surface of a sphere of radius a, $\vec{n}$ being the units normal to the surface.

4. Answer any **one** question: $10\times1=10$

a) i) Show that the equation of the right circular cylinder whose guiding curve is the circle through the points (1,0,0), (0,1,0) and (0,0,1) is $x^2+y^2+z^2-yz-zx-xy=0$

ii) A tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$ meets the co-ordinate axes in P Q, R. Find the locus of the centroid of the triangle PQR. 5+5

b) i) Show that the vector function $\vec{F} = (2x - yz)\hat{i} + (2y - zx)\hat{j} + (2z - xy)\hat{k}$ is irrotational.

ii) If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$ then find the value of $\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}$.

iii) Show that curl $\left(\text{curl}\,\vec{f}\right) = \vec{\nabla}\left(\vec{\nabla}.\vec{f}\right) - \nabla^2\vec{f}$.

4+3+3

c) i) Apply Stokes' theorem to evaluate $\int_{\Gamma}(\sin z dx - \cos x dy + \sin y dz)$ where Γ is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1,\ z = 3$.

ii) Find the equation of the straight lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. Find also the angles between the line. 5+5

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