U.G. 5th Semester Examination - 2020

MATHEMATICS

Course Code : BMTMDSRT-1 & 2 (DSE 1 & 2)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

This question papers contains both DSE 1 & 2. Students are thereby instructed to answer DSE paper out of these two (DSE 1 & DSE 2) as he/she opted for.

Course Title : Linear Programming

Course Code : BMTMDSRT-1 (DSE 1)

1. Answer any **ten** questions: 1×10=10

a) Express $\vec{\alpha}=(5, 9)$ as a linear combination of the vectors $\vec{\beta}=(1, 2)$ and $\vec{\gamma}=(2, 3)$.

b) Find whether the following vectors are linearly dependent or independent:

$$(1, 5, 7), (4, 0, 6), (1, 0, 0).$$

c) How many basic solutions will be possible for a system of two equations with three variables?

d) State the condition of optimality in simplex algorithm.

e) Verify whether the set $S=\{(x, y): x+2y=5\}$ is a convex set or not.

f) Let B be the basic matrix of the system AX=b, $X \geq 0$ and X_B be the basic solution. Then what is the relation between B, X_B and b?

g) Write down the convex combination of two vectors.

h) Find the dual of the LPP:

$$\text{Maximize } z=3x_1+2x_2$$
$$\text{subject to } 3x_1+4x_2 \leq 22$$
$$2x_1+3x_2 \leq 16$$
$$x_1, x_2 \geq 0$$

i) Write down the condition for existence of a feasible solution in a transportation problem.

j) Is it possible to solve an unbalanced transportation problem?

k) To obtain at least one zero in each row or column of the cost matrix in Hungarian method what is done?

l) Define optimal strategy in game theory.

m) Find the saddle point of the following game:

	B_1	B_2	B_3
A_1	15	2	3
A_2	6	5	7
A_3	−7	4	0

n) Find the position of the point (4, 1, 0, 1) with respect to the hyperplane $2x_1+x_2+3x_3+x_4=10$.

o) Write down a restriction imposed on a traveller in travelling salesman problem.

2. Answer any **five** questions: $2\times5=10$

a) Sketch graphically the feasible region, if any, for the following:

$$x_1 - x_2 \geq 0,\ 2x_1 + 3x_2 \leq 6;\ x_1,\ x_2 \geq 0$$

b) Find a basic solution of the equations:

$$2x_1+3x_2+x_3=8$$
$$x_1+2x_2+2x_3=5$$

c) Find the dual of the following problem:

Minimize $z = x_1 + 5x_2$

subject to $3x_1 + 4x_2 \leq 6$

$x_1 + 3x_2 \leq 2$

$x_1, x_2 \geq 0$

d) State complementary slackness theorem in duality theory.

e) Obtain an initial BFS of the TP using N-W corner rule.

	D_1	D_2	D_3	a_i
O_1	3	2	5	6
O_2	9	1	2	10
O_3	4	3	1	12
b_j	9	16	3	

f) Formulate mathematically an assignment problem.

g) Prove that the following pay-off matrix has no saddle point:

		B I	B II	B III
A	I	4	6	2
A	II	1	4	6
A	III	3	2	6

h) Define an extreme point of a convex set.

3. Answer any **two** questions: 5×2=10

a) Solve graphically:

Maximize $z = 4x_1 + 7x_2$

subject to $2x_1 + x_2 \le 1000$

$10x_1 + 10x_2 \le 6000$

$2x_1 + 4x_2 \le 2000$

$x_1, x_2 \ge 0$

b) Solve the following transportation problem by Vogel's approximation method:

	D_1	D_2	D_3	D_4	a_i
O_1	7	10	14	8	30
O_2	7	11	12	6	40
O_3	5	8	15	9	30
b_j	20	20	25	35	

c) Solve the following 2×3 game graphically:

	Player B		
Player A	1	3	11
	8	5	2

4. Answer any **one** question: 10×1=10

a) i) Reduce the following matrix game into 2×2 matrix game:

2	2	1	–2	–3
4	3	4	–2	0
5	1	2	5	6

Hence solve the game.

ii) Solve the following assignment problem with the following cost matrix:

	M_1	M_2	M_3	M_4
J_1	9	6	6	5
J_2	8	7	5	6
J_3	8	6	5	7
J_4	9	9	8	8

(3+3)+4=10

b) i) Prove that the set of all convex combinations of a finite number of points is a convex set. Is it a convex polyhedron?

ii) Use dual simplex method to solve the LPP:

Maximize $z = -2x_1 - 3x_2 - x_3$

subject to $2x_1 + x_2 + 2x_3 \geq 3$

$3x_1 + 2x_2 + x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

(3+1)+6=10

c) i) Solve the following LPP:

Maximize $z = 3x_1 + 4x_2$

subject to $x_1 + x_2 \leq 10$

$2x_1 + 3x_2 \leq 18$

$x_1 \leq 8$

$x_2 \geq 6$

$x_1, \; x_2 \geq 0$

By solving its dual problem.

ii) x_1=2, x_2=4 and x_3=1 is a feasible solution to the system of equations

$$\begin{pmatrix} 2 & -1 & 2 \\ 1 & 4 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 18 \end{pmatrix}.$$

Reduce the feasible solution to a basic feasible one. (3+3)+4=10

Course Title : Mechanics-I

Course Code : BMTMDSRT-2 (DSE 2)

1. Answer any **ten** questions: 1×10=10

a) State Newton's laws of motion.

b) Define inertial frame of reference with examples.

c) Obtain the formula for the time rate of change of angular momentum about a fixed point.

d) What are the Galilean transformations?

e) What do you mean by degrees of freedom of a system?

f) State the postulates of 'special theory of relativity'.

g) What do you mean by potential energy of a system?

h) State the principles of conservation of linear momentum and energy for a system of particles.

i) On what dynamical principle does a jet plane moves?

j) Give the concept of 'absolute time' and 'absolute length'.

k) Show that the virtual work done by the reaction of the body sliding over a smooth surface is zero.

l) What are the physical significance of the eigenvalues and eigenvectors of the inertia matrix?

m) Can a rigid body be deformable?

n) What are principal axis? What is the form of inertia matrix with respect to principal axes?

o) State D'Alemberts principle for rigid bodies.

2. Answer any **five** questions: $2 \times 5 = 10$

a) Write down the dimensions of the following:

i) Angular velocity

ii) Gravitational constant

iii) Viscosity

iv) Radius of gyration

b) When is a body called strained? When is a body called deformable?

c) Is a frame rotating uniformly with respect to an inertial frame? Justify.

d) What is simple pendulum? Write down its kinetic and potential energies.

e) State the principle of transmissibility of force acting on a rigid body.

f) Show that Newton's second law of motion is invariant under Galilean transformation.

g) Show that the distance between two points is invariant in two inertial frames.

h) What do you mean by degrees of freedom of a system? Give example.

3. Answer any **two** questions: $5 \times 2 = 10$

a) i) What do you understand by a constraint in a dynamical system? Give examples.

ii) Give the concept of 'absolute time' and 'absolute length'. $(2+1)+(1+1)=5$

b) Find the moment of inertia of a right circular cylinder about an axis through its C.G. perpendicular to its axis. 5

c) State and prove D'Alembert's Principle. 5

4. Answer any **one** question: $10 \times 1 = 10$

a) State and prove the parallel axis theorem. $3+7=10$

b) A uniform sphere of radius a rolls down an inclined plane, rough enough to prevent any sliding. Discuss the motion and show that for pure rolling $\mu > \frac{2}{7}\tan\alpha$, where α is the inclination of the plane from the horizontal ground level and μ is the coefficient of friction. 6+4=10

c) A small insect moves along a uniform bar of mass equal to itself and of length 2a, the ends of which are constrained to remain on the circumference of a fixed circle whose radius is $\frac{2a}{\sqrt{3}}$. If the insect starts from the middle point of the bar and moves along the bar with relative velocity V, then show that the bar in time t will turn through an angle

$$\frac{1}{\sqrt{3}}\tan^{-1}\frac{Vt}{a}.$$ 10

————————