U.G. 5th Semester Examination - 2020

PHYSICS

Course Code : BPHSCCHC501

Course Title : Quantum Mechanics and Applications

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

a) The wave function of a particle moving in one dimension is given to be

$$\psi(x) = \sqrt{\frac{15}{a}}A\left(a^2 - x^2\right) \quad \text{for } -a \le x \ge 0$$
$$= 0 \quad \text{for } |x| > a$$

Find the value of A that will normalize $\psi(x)$.

b) What is Bohr Magneton?

c) Explain zero-point energy.

d) Show that $\left[y,\ p_y\right] = i\hbar$.

e) What is the electron configuration of the ground state of sodium atom?

f) Compute the probability current density for the unnormalized wave function $\psi = e^{ikx}$.

g) What is Pauli's Exclusion principle?

h) Explain L-S coupling.

i) Can a real wave function have a current?

j) Why is it important to study the harmonic oscillator problem?

k) Define eigenvalues and eigenfunctions of an operator.

l) If $\psi(x, t)$ is a Schrodinger's wavefunction, prove that $\psi^*(x, t)\,\psi(x, t)$ is necessarily real, and either positive or zero.

m) What is anomalous Zeeman effect?

n) Evaluate the commutator $[L_z, Z]$.

o) What is the differential operator in quantum mechanics that corredpond to p_x, the x-component of linear momentum of a particle?

2. Answer any **five** questions: 2×5=10

a) The wave function of a particle of time t is given as

$$\psi(x, t) = a_1\phi_1(r)e^{--i\varepsilon_1 t/\hbar} + a_2\phi_2(r)e^{--i\varepsilon_2 t/\hbar}$$

where ϕ_1 and ϕ_2 are two normalized eigenfunctions with eigenvalues ε_1 and $\varepsilon_2\left(\varepsilon_1 \neq \varepsilon_2\right)$; a_1 and a_2 are constants. Does $\psi(x, t)$ represent a stationary state? Justify your answer.

b) State and explain Stark effect.

c) Write down three basic postulates of wave mechanics and explain them.

d) Define expectation value of an observable. Using this definition show that $\frac{d}{dt}\langle x\rangle = \frac{1}{m}\langle P_x\rangle$ with symbols having usual meaning.

e) A particle is free to move within a potential box with rigid walls extending from $x=-\frac{l}{2}$ to $x=+\frac{l}{2}$. Find the wavefunctions.

f) Energy eigenfunction for a particle in a box of length L and with infinitely hard walls, is given by

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}.$$

Calculate the expectation value of x and p_x.

g) State and explain Hund's rule.

h) A particle on the x-axis has the wavefunction $\psi = cx^2$ between x=0 and x=2. Find the probability that the particle can be found between x=0.50 and x=0.60.

3. Answer any **two** questions: 5×2=10

a) Starting from the time-dependent Schrodinger equation, deduce the time-independent equation for three-dimensional motion. What are stationary states? 4+1

b) Taking normalized wavefunction of hydrogen atom in the ground state as:

$$\psi(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}, \text{ (}a_0\text{=first Bohr radius)}$$

Find an expression for the probability of finding an electron between r and r+dr. Find also the value of r at which this probability is maximum. How many electrons are allowed in states with principal quantum number n=2? Give reasons for your answer. 2+1+1+1

c) What is the ground state electronic configuration of potassium, which is an alkali metal with z=19? What are the L, S and J quantum numbers of the states? Describe the Zeeman effect of the ground state quantitatively. Draw pictorially the energy levels of a particle in a harmonic oscillator potential. 1+1+2+1

____