Raghunathpur College
Internal Examination-2020
Semester-III, Mathematics(Hons)
Paper-BMTMCCHT302
Algebra-II

Answer any **one** question $1 \times 10 = 10$

1. (i) Suppose $(G,\circ)$ be a group and H be a non empty subset of G. Prove that H is a subgroup of $(G,\circ)$ if and only if $a \in H, b \in H \Rightarrow a \circ b^{-1} \in H$.
   (ii) Show that the subset $Z(G) = \{a \in G: ax = xa, \ \forall\, x \in G\}$ of a group G is a subgroup of G.
   (iii) Let G be a group and H be a subgroup of G. Prove that for any two elements $a, b \in G$, either $aH = bH$ or $aH \cap bH = \varphi$, where aH, bH are two left cosets of H in G. 4+2+4=10
2. (i) If an abelian group G of order 10 contains an element of order 5, prove that G must be a cyclic group.
   (ii) Prove that the ring $\mathbb{Z}[i] = \{a + ib: a, b \in \mathbb{Z}\}$, the ring of Gaussian integers, is an integral domain.
   (iii) If F be a field, prove that the non-zero elements of F form an abelian group with respect to multiplication. 3+3+4=10