RAGHUNATHPUR COLLEGE

UG 5th Semester Internal Examination 2020

MATHEMATICS

Course Code: BMTMCCHT502

Course Title: Metric Space & Complex Analysis

Full Marks: 10 ***Time: 45 Minutes***

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meanings.

Answer any one question given below **1×10=10**

1. a) Show that the function $f(z) = \frac{x^3(1+i) - y^3(1-i)}{|z|^2}$, for $z \neq 0$

   $= 0$ for $z = 0$

   is continuous and satisfies Cauchy-Riemann equations at $z = 0$ but $f'(0)$ does not exist.

   b) Prove that every separable metric space is second countable.

   c) Let (X, d) be a metric space with at least two points. If a and b are two distinct points of X, then there exists two disjoint open spheres with centres a and b respectively.

   3+4+3

2. a) If z_1 and z_2 are the roots of the quadratic equation $\alpha z^2 + 2\beta z + \gamma = 0$, then prove that

   $$|z_1| + |z_2| = \frac{1}{|\alpha|}\{|-\beta + \sqrt{\alpha\gamma}| + |-\beta - \sqrt{\alpha\gamma}|\}$$

   b) Let (X, d) be a metric space and $\{x_n\}$ is a sequence of elements of X converging to $x \in X$. Then for any $\alpha \in X$, the sequence $\{y_n\}$, where $y_n = d(x_n, \alpha)$, converges to $d(x, \alpha)$ in the real line.

   c) Show that the diagonal $\{(x, x) : x \in X\}$ is closed in the product metric space $X \times X$.

   4+4+2

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