2020
PHYSICS
[HONOURS]
Paper : III

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** from the following: 2×10=20

a) Distinguish between drift current and diffusion current.

b) Define generalised co-ordinates and cyclic co-ordinates.

c) State whether the constraint is holonomic or non-holonomic with suitable reasons: (i) A simple pendulum. (ii) A pendulum with variable length.

d) Find the number of degrees of freedom for the following systems:

(i) A triatomic gas molecule in an enclosure with constant interatomic distance (ii) Two particles connected by a massless spring constrained to move in space.

e) What are the advantages of Hamiltonian formulation over Lagrangian one?

f) What do you mean by 'spatial coherence' and 'temporal' coherence'?

g) In the Fresnel biprism arrangement, the base angle of the biprism are kept small— why?

h) If the vector potential $\vec{A} = \left(x^2 + y^2 - z^2\right)\hat{j}$ at position (x, y, z), find the magnetic field at $(1, 1, 1)$.

i) Show that the Hamiltonian remains conserved for a system where the Lagrangian does not explicitly depend on time.

j) Explain Huygen's principle of wave propagation. Deduce laws of refraction of plane waves from this principle.

k) What are spontaneous and stimulated co-efficients in case of laser emission?

l) Can an electrostatic field vector have the following form $\vec{E} = x\hat{i} - y\hat{j}$? Give reasons.

m) Derive the dimension of electrical resistance in terms of the length, mass, time and current dimension.

n) A solenoid has $L = 53\,mH$ and $R = 0.37\,\Omega$ is connected to a battery. How long the current will take to reach half its final equilibrium value?

o) Explain why an extended source is necessary to observe colour in thin films.

p) In Newton's ring experiment, if a liquid of refractive index μ is introduced between the lens and the glass plate, how will the radii change?

GROUP–A

(Electrostatics)

Answer any **two** questions: 5×2=10

2. Two point charges $-q_1$ and $+q_2 (q_1 \neq q_2)$ are placed at (0,0,0) and (a,0,0) respectively. Show that equipotential surface corresponding to zero potential is a sphere. Find the centre and radius of this sphere. What happens when the magnitude of the charges are identical? 5

3. Explain the method of electrical image in solving electrostatic problem. Prove Gauss's theorem from Coulomb's law. Hence prove $\vec{\nabla}.\vec{E} = \frac{e}{\epsilon_0}$

where the symbols have their usual meaning. 2+3=5

4. i) A neutral water molecule in its vapour state has an electric dipole moment of magnitude 6.2×10^{-30} c-m. How far apart are the molecules centre of positive charge and negative charge? How much work must an external agent do to turn the molecule end to end in an electric field of 1.5×10^{4} N/C, starting from its fully aligned position.

ii) Derive Coulomb's law from Gauss's law. 1+2+2=5

5. i) Define Polarization Vector $\vec{P}$ for a dielectric substance.

ii) Show that the electric field produced by a polarized dielectric can be given by the contributions from a bound surface charge density $\sigma_b = \vec{P}.\hat{n}$ and a volume charge density $\rho_b = -\vec{\nabla}.\vec{P}$, where $\vec{P}$ is the polarization and $\hat{n}$ is the unit normal to the surface. 1+4=5

Answer any **one** question: 10×1=10

6. i) Write down the Laplace's equation and its connection with uniqueness theorem in electrostatics.

ii) A circular wire of radius 'a' has linear charge density $\lambda = \lambda_0 \cos^2\theta$ where θ is the angle with respect to a fixed radius. Calculate (a) total charge (b) potential at the centre.

iii) Two similar point charges q are kept separated by a distance '2d' in air. Now an earthed conducting sphere of radius 'a' is placed midway between them. If d>>a, show that the introduction of the sphere just neutralises the repulsive force existing between the point charges if $d \approx 8a$.

1+2+4+3=10

7. i) The distance between the plates of a parallel plate air condenser is d. A dielectric slab of thickness x is introduced in the air gap. Show that the capacity of the condenser will be doubled if the dielectric constant of the slab is $k = \frac{2x}{(2x-d)}$.

ii) A point charge of mass m is released from rest at a distance d from an infinite grounded conducting plate. Show that the time taken by the charge to hit the plate is $t = \frac{\pi}{q}\sqrt{\frac{md^3}{2}}$.

iii) Using Gauss's law prove that any charge placed on a conductor must lie entirely on its surface. 3+5+2=10

GROUP–B

(Electrodynamics-I)

Answer any **two** questions: 5×2=10

8. i) Find the average value of ac current represented by $I = I_0 \sin\omega t$ for $0 \le \omega t \le \pi$; $I = -I_0 \sin\omega t$ for $\pi \le \omega t \le 2\pi$.

ii) Find the r.m.s value of a zero mean sinusoidal voltage of frequency 100Hz and peak value $\sqrt{2}$ volts. What will be the value of that quantity if the frequency be changed to 200Hz? 2+3=5

9. What is a magnetic circuit? Establish a relation between the magnetomotive force, the reluctance and the magnetic flux. Discuss the analogy between electric and magnetic circuits.

1+2+2=5

10. i) Verify that the vector potential $\vec{A}$ due to a uniform magnetic induction $\vec{B}$ is given by $\vec{A} = -\frac{1}{2}(\vec{r} \times \vec{B})$.

ii) Find the magnetic moment of a circuit consisting of two mutually perpendicular

squares, both of them have equal side length 'a' and one lying on xy-plane and other on xz-plane. The current is flowing in the anti-clockwise direction in both the loops. 3+2=5

11. i) Two magnetic media of relative permeabilities μ_1 and μ_2 are separated by a plane interface. Show that the angles between the normal to the boundary and the field lines satisfy the relation $\mu_1 \tan\theta_2 = \mu_2 \tan\theta_1$.

ii) Write down Bio-Savart law for surface and volume currents. 3+2=5

Answer any **one** question: 10×1=10

12. i) Calculate the magnetic vector potential $\vec{A}$ due to a long straight wire carrying current I and away from it. Find the value of the magnetic induction from the expression of $\vec{A}$.

ii) A square loop of side 'a' is placed in the plane of a long, straight wire carrying current I. The nearest side of the loop is at a distance r from the wire. Find the magnetic flux through the loop. Calculate the e.m.f. generated in the loop if the loop is pulled (i) away from the wire (ii) parallel to the wire. (3+2)+(3+2)=10

13. i) Show that the equation of continuity is given by $\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$, where $\vec{J}$ and ρ have their usual meaning.

ii) Discuss the theory and action of an ideal transformer. Obtain expressions for effective resistance and effective inductance of the primary coil. 3+5+1+1=10

GROUP–C

(Classical Mechanics-II)

Answer any **two** questions: 5×2=10

14. What do you mean by degrees of freedom? Using the relation between Lagrangian and Hamiltonian of a system derive Hamilton's equation of motion. 2+3=5

15. What is meant by virtual displacement? State the principle of virtual work and use it to establish D'Alembert's principle. 5

16. Show that if q_k is a cyclic co-ordinate in Lagrangian, it will also be a cyclic co-ordinate in Hamiltonian.

17. Find the Lagrangian of the following systems:

i) A mass 'm' connected on either side with springs having spring constant k_1 and k_2 and the free ends of springs are tied to rigid support.

ii) An electrical circuit comprising an inductance *L* and capacitance *C*, charged to q Coulomb and current 'i' flowing in the circuit. $2\frac{1}{2}\times2=5$

Answer any **one** question: $10\times1=10$

18. Prove that for a conservative system in which the Lagrangian is not an explicit function of time, the Hamiltonian is conserved. Find the Lagrangian and Hamiltonian for a particle in a rotating frame. 4+3+3=10

19. i) Prove that for Scleronomic System with velocity independent potential, Hamiltonian is identical to the total mechanical energy.

ii) Derive Newton's second law from Hamilton's principle.

iii) Write down the Hamiltonian for a simple pendulum. Derive an expression for the phase trajectory of the particle and identify it. 3+3+(1+3)=10

GROUP–D

(Physical Optics)

Answer any **two** questions: $5\times2=10$

20. i) What is a quarter wave plate? Discuss how it can be used to produce circularly and elliptically polarized light.

ii) What do you mean by positive and negative crystals? 4+1=5

21. i) Indicate how polarized light is obtained using the phenomenon double refraction.

ii) An unpolarized light of intensity I_0 is passed through two Nicol prisms with their Principal Sections at 45^o to each other. What is the intensity of the transmitted light? 3+2=5

22. i) How 'ghost lines' appear in a grating diffraction pattern?

ii) Find the least width that a diffraction grating must have to resolve two Sodium D-lines $(\lambda_1=5890\text{Å}, \lambda_2=5896\text{Å})$ in the second order. The number of lines per cm. of the grating is 820. 2+3=5

23. i) Define the dispersive power of a grating and find an expression for it.

ii) A left circularly polarized light is passed through a half-wave plate. What will be the state of polarization of the emergent light? Explain. 1+1+3=5

Answer any **one** question: 10×1=10

24. i) What is a zone plate? Compare it with a lens.

ii) What is Kerr effect? Name a transparent substance which shows the effect.

iii) A plane polarised light of wavelength 600nm changes to a circularly polarised light on passing through a quartz crystal cut parallel to optical axis. Calculate the minimum thickness to produce such effect. Given $n_e - n_o = 0.005$. (2+2)+(2+1)+3=10

25. i) Calculate the coherence time and spectral width $\Delta\lambda$ for a quasi monochromate source of mean wavelength 6438 Å and coherence length 30 cm.

ii) Write down the equations representing the following waves:

A. A wave with right handed circular polarization.

B. A wave with right handed elliptic polarization and with its major axis along the y-direction, this axis being twice the minor axis.

iii) State of population inversion is sometimes called negative temperature – Explain.

iv) Give one example of a 3-level laser and one example of a 4-level laser.

3+(2+2)+2+1=10

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