2020
B.C.A.
[HONOURS]
(Mathematics)
Paper : BCA-201

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **eight** of the following: 2×8=16

   a) State Leibnitz's Theorem.

   b) Find the order and degree of the differential equation

   $$\left(\frac{d^3y}{dx^3}\right)^{2/3} = x^2\frac{dy}{dx}.$$

   c) Can you apply Rolle's theorem to $f(x)=\tan x$ in $[0,\pi]$? Give reason for your answer.

   d) If f(x) is an odd function in $[-\pi,\pi]$ then show that $\int_{-\pi}^{\pi} f(x)dx = 0$.

   e) Evaluate $\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$.

   f) Find the particular integral of $(D^2-D-6)y = x$.

   g) What do you mean by the convergence of a series?

   h) Show that the infinite series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots$ is convergent.

   i) Evaluate $\int_{-\pi/2}^{\pi/2}|\cos x|dx$.

   j) State the rule for integration by parts for the product of two functions.

   k) Show that the sequence $\left\{\frac{n+1}{n}\right\}$ is bounded.

   l) Find the integrating factor of the linear differential equation $\cos x\frac{dy}{dx}+y\sin x = 1$.

2. Answer any **four** from the following : 4×4=16

   a) Let f(x) be a function defined as

   $$f(x)=\begin{cases} 3+2x & \text{for } -\frac{3}{2}\le x<0 \\ 3-2x & \text{for } 0\le x<\frac{3}{2} \\ -3-2x & \text{for } x\ge\frac{3}{2} \end{cases}$$

Show that f(x) is continuous at $x=0$ and discontinuous at $x=\frac{3}{2}$. 4

b) Given $f(x)=ax^2+bx+c$, show that

$$\lim_{h\to 0}\left\{\frac{f(x+h)-f(x)}{h}\right\}=2ax+b.$$

c) If $y=x^{n-1}\log^x$ then show that $y_n=\frac{(n-1)!}{x}$.

4

d) State and prove Taylor's theorem with Lagrange's form of remainder. 4

e) Show that

$$x-\frac{x^2}{2}<\log(1+x)<x-\frac{x^2}{2(1+x)}, x>0 \quad 4$$

f) If $u=\tan^{-1}\frac{x^3+y^3}{x^2+y^2}$, applying Euler's theorem to show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u.$$

4

3. Answer any **four** from the following : 4×4=16

a) Evaluate the following:

i) $\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx$

ii) $\int \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)dx$

b) Prove that

i) $\int_a^b f(x)\,dx=\int_a^c f(x)\,dx+\int_c^b f(x)\,dx$

ii) $\int_0^{na} f(x)\,dx=n\int_0^a f(x)\,dx$, if $f(x)=f(a+x)$

c) Prove that $\lim_{n\to\infty}\frac{1^m+2^m+3^m+...+n^m}{n^{m+1}}=\frac{1}{m+1}$

4

d) Find from the definition, the value of $\int_0^1 x^2 dx$.

4

e) Show that $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$ is convergent and find its value.

f) i) State Fundamental theorem of Integral Calculus.

ii) Evaluate the improper integral $\int_0^1 \frac{dx}{x^{2/3}}$.

2+2=4

4. Answer any **four** from the following : 4×4=16

a) Obtain the differential equation of all circles each of which touches the axis of x at the origin.

b) Solve : $x\frac{dy}{dx}+y=x^2y^2$.

c) Solve the equation $y=px+\sqrt{1+p^2}$, $p=\frac{dy}{dx}$ and obtain the singular solution.

d) Solve $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$. 4

e) Solve $y=px+\frac{a}{p}, p\equiv\frac{dy}{dx}$ and obtain the singular solution. 4

f) Solve in particular case

$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$, when $x=0, y=3$ and $\frac{dy}{dx}=0$. 4

5. Answer any **four** from the following : 4×4=16

a) Prove that the sequence $\left\{\frac{3n-1}{n+2}\right\}$ is monotone increasing and bounded.

b) Prove that the sequence $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}.....$, converges to 2.

c) State Cauchy's general principle of convergence and using this show that the sequence $\{f_n\}$, where $f_n=1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$ is not convergent. 2+2=4

d) Show that the given series is divergent $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$.

e) Show that $1-\frac{1}{4.3}+\frac{1}{4^2.5}-\frac{1}{4^3.7}+...$ is convergent. 4

f) i) Examine the convergence of the series $\sum_{n=1}^{\infty}\sin\frac{1}{n}$.

ii) Show that the series $\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+...+\sqrt{\frac{n}{2(n+1)}}+...$ does not converge. 2+2=4

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