U.G. 6th Semester Examination - 2020

MATHEMATICS

Course Code : BMTMGERT10A

Course Title : Basics of Higher Mathematics-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1\times10=10$

a) If 'a' be a fixed element of a group $(G, *)$ then solve the equation $a*(x*a)=a$.

b) Show that the multiplicative inverse of a non-zero element in a field is unique.

c) Find the direction cosines of the normal of the plane $2x+4y-6z=5$.

d) Find the angle between the pair of straight lines represented by the equation

$$2x^2+3xy-2y^2=0.$$

[Turn Over]

e) Find the equation of the plane passing through $P(a, b, c)$ and perpendicular to OP, where O is the origin.

f) Eliminate a and b from $y=a+b\log_e x$ and find the order of the D.E.

g) Convert $(6x-5y+4)dy+(y-2x-1)dx=0$ into a homogeneous equation.

h) Find the centre and diameter of the sphere

$$x^2+y^2+z^2-2x+4y-6z=2$$

i) For what value of λ,

$$3x^2+\lambda xy-5y^2+2x+2y=0$$

will be a pair of straight lines?

j) Transform the equation $r^{\frac{1}{2}}\cos\frac{\theta}{2}=a^{\frac{1}{2}}$ to Cartesian co-ordinates.

k) Find the differential equation of all circles touching the X-axis at the origin.

l) Find the nature of the conic $\frac{l}{r}=4-5\cos\theta$.

m) Define abelian group.

n) Prove that if in a ring, the unit element exists, then it is unique.

o) With the help of integration find the circumference of a circle of radius a.

2. Answer any **five** questions: 2×5=10

a) If $a, b \in G$, then prove that $(a.b)^{-1} = b^{-1}.a^{-1}$, where $(G, \bullet)$ is a group.

b) Show that the set of even integers does not form a field w.r.to arithmetic addition and multiplication.

c) Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} cos^n x \, dx$.

d) Let a, b, c be arbitrary elements of a group $(G, *)$. If $a*b = a*c$, then prove that $b = c$.

e) The co-ordinate axes are rotated through an angle 60^o. If the transformed co-ordinates of a point are $\left(2\sqrt{3}, -6\right)$, find its original co-ordinate.

f) Find the co-ordinates of the point in which the straight line $\frac{x-1}{3} = \frac{y+2}{-1} = \frac{z}{4}$ intersects the plane $4x + y + z = 2$.

g) Obtain the integrating factor of $\frac{dy}{dx} + \frac{1-2x}{x^2} y = 1$.

h) Find the area of a quadrant of an ellipse.

3. Answer any **two** questions: 5×2=10

a) i) Show that the straight line $\frac{l}{r} = a\cos\theta + b\sin\theta$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$, if $(a-e)^2 + b^2 = 1$.

ii) In a ring $(\mathrm{R}, +, \bullet)$ prove that for any $a \in R$, $a.0 = 0.a = 0$, where 0 is the additive identity in R. 3+2

b) i) Solve: $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0$.

ii) Find the equation of the plane through the line of intersection of the planes $x - 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + z = 2$. 2+3

c) i) Examine, is the set $\{1, i, -1, -i\}$ forms a group with respect to multiplication.

ii) Examine whether $\frac{1}{y^2}$ is an integrating factor of the differential equation $y(1+xy)dx - xdy = 0$. 3+2

4. Answer any **one** question: 10×1=10

a) i) Find the nature of the conic $x^2+4xy+4y^2+4x+y-15=0$ and reduce it to its canonical form.

ii) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, where n is a positive integer, then show that $I_{(n+1)} + I_{(n-1)} = \frac{1}{n}$.

iii) Show that the set $\{1, \omega, \omega^2\}$, ω being an imaginary cube root of unity, forms a group with respect to multiplication.

5+3+2

b) i) Find the shortest distance between the straight lines $\frac{x+1}{1} = \frac{y-4}{3} = \frac{z+3}{4}$ and the x-aixs.

ii) Find the general solution of $y = 2px + y^2p^3$, $p = \frac{dy}{dx}$.

iii) Show that the set of all integers is a commutative ring but not a field.

4+3+3

c) i) Show that $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 1 & 2 & 6 & 5 \end{pmatrix}$ is an even permutation.

ii) Show that the area of the triangle formed by the lines $lx+my+n=0$ and $ax^2+2hxy+by^2=0$ is $\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}$.

iii) Solve: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \sin x$, given that $y=0$, $\frac{dy}{dx}=0$, when $x=0$. 2+4+4

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