U.G. 6th Semester Examination - 2020

MATHEMATICS

Course Code : BMTMDSHT4

Course Title : Probability and Statistics

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Define sample space.

b) A die is thrown. Find the probability of getting 'multiple of three'.

c) What are the values of the random variable in case of tossing a coin?

d) Show that $\mathrm{Var}(X)=E(X^2)-\{E(X)\}^2$.

e) For Poisson distribution if its probability mass function is f_i, show that $\sum_i f_i = 1$.

f) For continuous case, write down the marginal density functions of two variables for joint probability distribution of two variables.

g) Define correlation coefficient of two random variables by specifying the terms.

h) What is the relation between conditional mean and regression function?

i) State Tchebycheff's Inequality.

j) Write down the formula for two Beta coefficients.

k) Give the names of types of population.

l) Write down the density function of χ^2 distribution mentioning the range of the variable.

m) When an estimation is called biased? What is the measure of bias?

n) Show that in estimating the mean of a normal population, sample mean is more efficient than sample median.

o) Define critical region in connection with testing of hypothesis.

2. Answer any **five** questions: $2 \times 5 = 10$

a) If A and B are two independent events, then prove that $\overline{A}$ and $\overline{B}$ are also independent.

b) If
$$f(x) = \begin{cases} kx^2 e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$
find the value of the constant k so that f(x) is a probability density function.

c) Find the characteristic function of the Binomial distribution.

d) Let the joint distribution of X and Y be given by the p.d.f.
$$f(x, y) = \begin{cases} x + y, & \text{if } 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$
Find E(XY) and E(X+Y).

e) A random variable X has probability density function $f(x) = 12x^2(1-x),\ 0 < x < 1$.

Compute $P(|X - m| \geq 2\sigma)$.

f) If m_r and α_r denote the r-th central moments of x and of the standardised variable $z = \frac{x - \overline{x}}{\sigma}$ respectively, where σ is the S.D. of x, then show that $\alpha_r = \frac{m_r}{\sigma^r}$.

g) Find the maximum likelihood estimate of the parameter λ of the distribution with p.d.f.
$$f(x) = \lambda \alpha x^{\alpha - 1} e^{-\lambda x^{\alpha}},\ x > 0$$
using a sample size n, assuming that α is known.

h) Write a short note on Interval Estimation.

3. Answer any **two** questions: $5 \times 2 = 10$

a) If X be a standard normal variate, find the probability density function of Y, where $Y = \frac{1}{2} X^2$.

b) The joint probability density function of the random variable x and y is
$$f(x, y) = \begin{cases} k(1 - x - y), & x \geq 0,\ y \geq 0,\ x + y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$
where, k is a constant. Find

i) the value of k

ii) the marginal probability density functions

iii) the mean value of y when $x = \frac{1}{2}$.

c) Ten individuals are chosen at random from a normal population with mean m and standard deviation σ and their heights in inches are found to be

63, 66, 63, 67, 68, 69, 70, 71, 72 and 71.

Find 95% confidence interval for the parameter m. Given $P(t > 2.262) = 0.025$ for 9 degrees of freedom.

4. Answer any **one** question: $10\times1=10$

a) i) If the joint distribution of X and Y is the general bivariate normal distribution and

$$U=\frac{X-m_x}{\sigma_x},\ V=\frac{1}{\sqrt{1-\rho^2}}\left\{\frac{Y-m_y}{\sigma_y}-\rho.\frac{X-m_x}{\sigma_x}\right\}$$

then prove that U and V are independent standard normal variates. 5

ii) Given $\Sigma x = 56$, $\Sigma y = 40$, $\Sigma x^2 = 524$, $\Sigma y^2 = 256$, $\Sigma xy = 364$, n=8. Find the equations of the regression lines of x on y and of y on x. 5

b) i) Find the moment generating function of the normal distribution. Hence obtain mean. 5

ii) Find out the skewness and kurtosis of the series by the method of moments:

Measurement:	0-10	10-20	20-30	30-40
Frequency:	1	3	4	2

5

c) i) If X and Y be correlated and U and V be defined by $U = X\cos\alpha + Y\sin\alpha$, $V = Y\cos\alpha - X\sin\alpha$, then prove that U and V will be uncorrelated if $\tan 2\alpha = \dfrac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}$. 5

ii) If $X_1, X_2, ..., X_n$ be a random sample from a normal (μ, σ^2) distribution, then show that $\dfrac{(n-1)S^2}{\sigma^2}$ is a χ^2 distribution with $(n-1)$ degrees of freedom where S^2 is the sample variance. 5

—————